Chapter 1

Line and Angle Relationships

1.7 The Formal Proof of a Theorem

The Formal Proof of a Theorem

Statements that follow logically from known undefined terms, definitions, and postulates are called *theorems*.

To understand the formal proof of a theorem, we begin by considering the terms *hypothesis* and *conclusion*.

The hypothesis of a statement describes the given situation (Given), whereas the conclusion describes what you need to establish (Prove).

The Formal Proof of a Theorem

When a statement has the form "If H, then C," the hypothesis is H and the conclusion is C.

Some theorems must be reworded to fit into "If . . . , then . . ." form so that the hypothesis and conclusion are easy to recognize.

Example 1

Give the hypothesis H and conclusion C for each of these statements.

a) If two lines intersect, then the vertical angles formed are congruent.

b) All right angles are congruent.

c) Parallel lines do not intersect.

d) Lines are perpendicular when they meet to form congruent adjacent angles.

Example 1 – *Solution*

a) As is H: Two lines intersect.

C: The vertical angles formed are congruent.

b) Reworded If two angles are right angles, then these angles are congruent.

H: Two angles are right angles.

C: The angles are congruent.

Example 1 – *Solution*

cont'd

c) Reworded If two lines are parallel, then these lines do not intersect.

H: Two lines are parallel.

C: The lines do not intersect.

d) Reordered When (if) two lines meet to form congruent adjacent angles, these lines are perpendicular.

H: Two lines meet to form congruent adjacent angles.

C: The lines are perpendicular.

THE WRITTEN PARTS OF A FORMAL PROOF

The Written Parts of a Formal Proof

The five necessary parts of a formal proof are listed in the following box in the order in which they should be developed.

ESSENTIAL PARTS OF THE FORMAL PROOF OF A THEOREM

1. *Statement:* States the theorem to be proved.
2. *Drawing:* Represents the hypothesis of the theorem.
3. *Given:* Describes the Drawing according to the information found in the hypothesis of the theorem.
4. *Prove:* Describes the Drawing according to the claim made in the conclusion of the theorem.
5. *Proof:* Orders a list of claims (Statements) and justifications (Reasons), beginning with the Given and ending with the Prove; there must be a logical flow in this Proof.

The Written Parts of a Formal Proof

The most difficult aspect of a formal proof is the thinking process that must take place between parts 4 and 5.

This game plan or analysis involves deducing and ordering conclusions based on the given situation.

One must be somewhat like a lawyer, selecting the claims that help prove the case while discarding those that are superfluous.

The Written Parts of a Formal Proof

In the process of ordering the statements, it may be beneficial to think in reverse order, like so:

The Prove statement would be true if what else were true?

The final proof must be arranged in an order that allows one to reason from an earlier statement to a later claim by using deduction (perhaps several times). Where principle *P* has the form "If *H*, then *C*," the logical order follows.

H: hypothesis $\longleftarrow$ statement of proof

P: principle $\longleftarrow$ reason of proof

$\therefore$ C: conclusion $\longleftarrow$ next statement in proof

The Written Parts of a Formal Proof

Theorem 1.6.1

If two lines are perpendicular, then they meet to form right angles.

Example 2

Write the parts of the formal proof of Theorem 1.6.1.

Solution:

1. State the theorem.

If two lines are perpendicular, then they meet to form right angles.

2. The hypothesis is H: Two lines are perpendicular.

Make a Drawing to fit this description. (See Figure 1.65.)

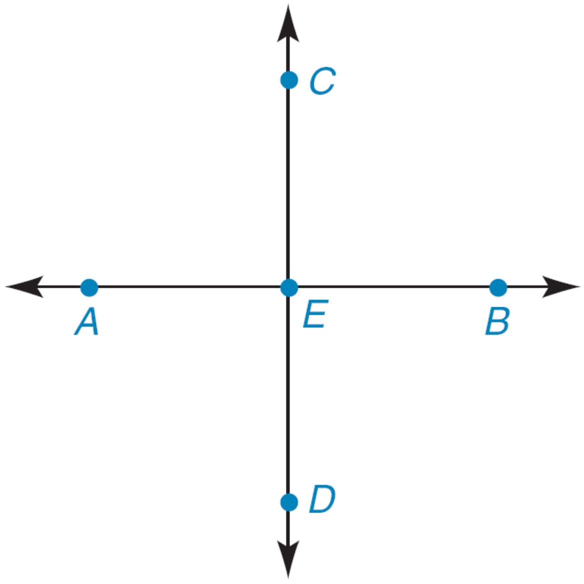

Figure 1.65

Example 2 – *Solution*

cont'd

3. Write the Given statement, using the Drawing and based on the hypothesis H: Two lines are $\perp$.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E

4. Write the Prove statement, using the Drawing and based on the conclusion C: They meet to form right angles.

Prove: $\angle AEC$ is a right angle.

5. Construct the Proof.

CONVERSE OF A STATEMENT

Converse of a Statement

The converse of the statement "If *P*, then *Q*" is "If *Q*, then *P*." That is, the converse of a given statement interchanges its hypothesis and conclusion.

Consider the following:

Statement: If a person lives in London, then that person lives in England.

Converse: If a person lives in England, then that person lives in London.

In this case, the given statement is true, whereas its converse is false. Sometimes the converse of a true statement is also true.

Converse of a Statement

The proof of a theorem is not unique! For instance, students' Drawings need not match, even though the same relationships should be indicated.

Certainly, different letters are likely to be chosen for the Drawing that illustrates the hypothesis.

Theorem 1.7.1

If two lines meet to form a right angle, then these lines are perpendicular.

Example 3

Give a formal proof for Theorem 1.7.1.

If two lines meet to form a right angle, then these lines are perpendicular.

Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at *E* so that $\angle AEC$ is a right angle (Figure 1.66)

Prove: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

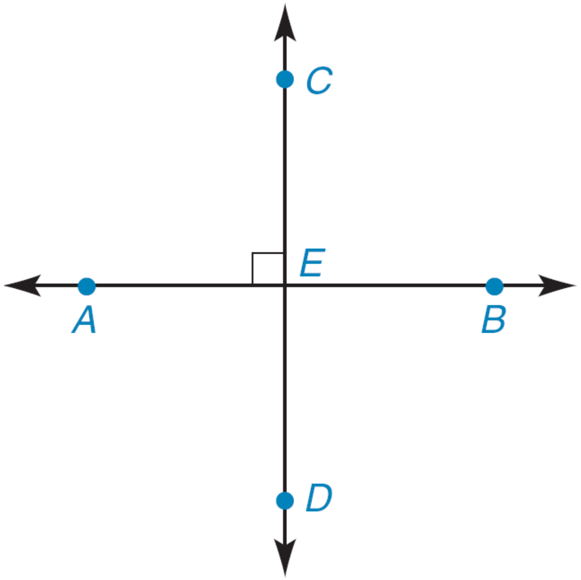

Figure 1.66

Example 3

cont'd

Proof:

Statements	Reasons
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect so that $\angle AEC$ is a right angle	1. Given
2. $m\angle AEC = 90$	2. If an $\angle$ is a right $\angle$, its measure is 90
3. $\angle AEB$ is a straight $\angle$, so $m\angle AEB = 180$	3. If an $\angle$ is a straight $\angle$, its measure is 180

Example 3

cont'd

	Statements	Reasons
	4. $m\angle AEC + m\angle CEB = m\angle AEB$	4. Angle-Addition Postulate
(2), (3), (4)	5. $90 + m\angle CEB = 180$	5. Substitution
(5)	6. $m\angle CEB = 90$	6. Subtraction Property of Equality
(2), (6)	7. $m\angle AEC = m\angle CEB$	7. Substitution

Example 3

cont'd

Statements	Reasons
8. $\angle AEC \cong \angle CEB$	8. If two $\angle$s have = measures, the $\angle$s are $\cong$
9. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$	9. If two lines form $\cong$ adjacent $\angle$s, these lines are $\perp$

Converse of a Statement

Theorem 1.7.2

If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.

Theorem 1.7.3

If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.

Converse of a Statement

Theorem 1.7.4

Any two right angles are congruent.

Theorem 1.7.5

If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.

Theorem 1.7.6

If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.

Converse of a Statement

Theorem 1.7.7

If two line segments are congruent, then their midpoints separate these segments into four congruent segments.

Theorem 1.7.8

If two angles are congruent, then their bisectors separate these angles into four congruent angles.