Chapter 1

Line and Angle Relationships

1.6 Relationships: Perpendicular Lines

Relationships: Perpendicular Lines

Informally, a **vertical** line is one that extends up and down, like a flagpole. On the other hand, a line that extends left to right is **horizontal.**

In Figure 1.59, ℓ is vertical and j is horizontal. Where lines ℓ and j intersect, they appear to form angles of equal measure.

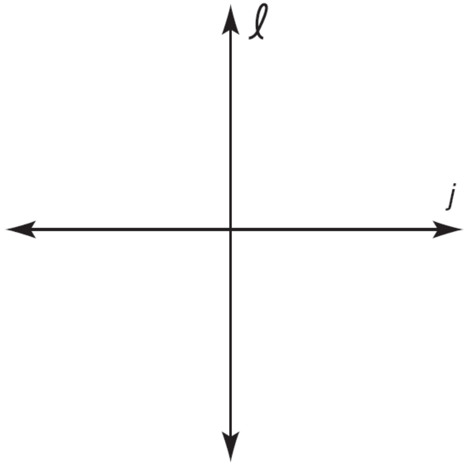

Figure 1.59

Relationships: Perpendicular Lines

Definition

Perpendicular lines are two lines that meet to form congruent adjacent angles.

Relationships: Perpendicular Lines

Perpendicular lines do not have to be vertical and horizontal. In Figure 1.60, the slanted lines *m* and *p* are perpendicular ($m \perp p$).

As in Figure 1.60, a small square is often placed in the opening of an angle formed by perpendicular lines.

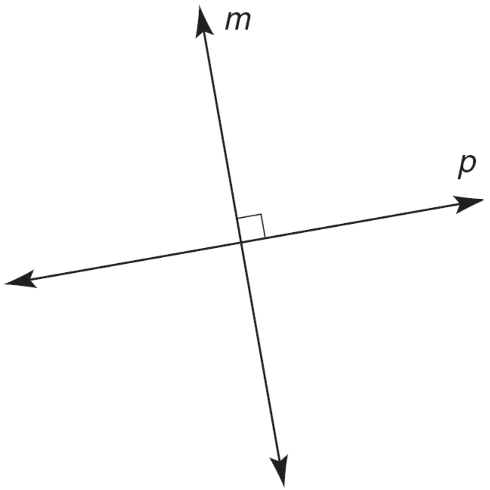

Figure 1.60

Relationships: Perpendicular Lines

Theorem 1.6.1

If two lines are perpendicular, then they meet to form right angles.

Example 1

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, intersecting at *E* (See Figure 1.61)

Prove: $\angle AEC$ is a right angle

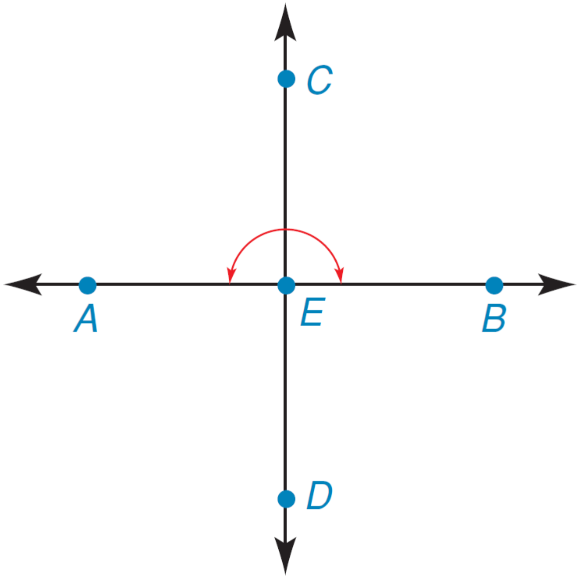

Figure 1.61

Example 1

cont'd

Proof:

Statements	Reasons
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, intersecting at *E*	1. Given
(1) 2. $\angle AEC \cong \angle CEB$	2. Perpendicular lines meet to form congruent adjacent angles (Definition)
(2) 3. $m\angle AEC = m\angle CEB$	3. If two angles are congruent, their measures are equal
4. $\angle AEB$ is a straight angle and $m\angle AEB = 180°$	4. Measure of a straight angle equals 180°

Example 1

cont'd

	Statements	Reasons
	5. $m\angle AEC + m\angle CEB = m\angle AEB$	5. Angle-Addition Postulate
(4), (5)	6. $m\angle AEC + m\angle CEB = 180°$	6. Substitution
(3), (6)	7. $m\angle AEC + m\angle AEC = 180°$ or $2 \cdot m\angle AEC = 180°$	7. Substitution
(7)	8. $m\angle AEC = 90°$	8. Division Property of Equality
(8)	9. $\angle AEC$ is a right angle	9. If the measure of an angle is 90°, then the angle is a right angle

RELATIONS

Relations

The relationship between perpendicular lines suggests the more general, but undefined, mathematical concept of **relation.**

In general, a relation “connects” two elements of an associated set of objects.

Relations

Table 1.8 provides several examples of the concept of a relation R.

TABLE 1.8

Relation R	**Objects Related**	**Example of Relationship**
is equal to	numbers	$2 + 3 = 5$
is greater than	numbers	$7 > 5$
is perpendicular to	lines	$\ell \perp m$
is complementary to	angles	$\angle 1$ is comp. to $\angle 2$
is congruent to	line segments	$\overline{AB} \cong \overline{CD}$
is a brother of	people	Matt is a brother of Phil

Relations

There are three special properties that may exist for a given relation R.

Where *a*, *b*, and *c* are objects associated with relation R, the properties consider one object (reflexive), two objects in either order (symmetric), or three objects (transitive).

For the properties to exist, it is necessary that the statements be true for all objects selected from the associated set.

Relations

These properties are generalized, and specific examples are given below:

Reflexive property: $a\text{R}a$ ($5 = 5$; equality of numbers has a reflexive property)

Symmetric property: If $a\text{R}b$, then $b\text{R}a$. (If $\ell \perp m$, then $m \perp \ell$; perpendicularity of lines has a symmetric property)

Transitive property: If $a\text{R}b$ and $b\text{R}c$, then $a\text{R}c$. (If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$; congruence of angles has a transitive property)

Example 2

Does the relation "is less than" for numbers have a reflexive property? a symmetric property? a transitive property?

Solution:

Because "$2 < 2$" is false, there is *no* reflexive property.

"If $2 < 5$, then $5 < 2$" is also false; there is *no* symmetric property.

"If $2 < 5$ and $5 < 9$, then $2 < 9$" is true; there is a transitive property.

Example 2 – *Solution*

cont'd

Note:

The same results are obtained for choices other than 2, 5, and 9.

Relations

Congruence of angles (or of line segments) is closely tied to equality of angle measures (or line segment measures) by the definition of congruence.

Relations

The following list gives some useful properties of the congruence of angles.

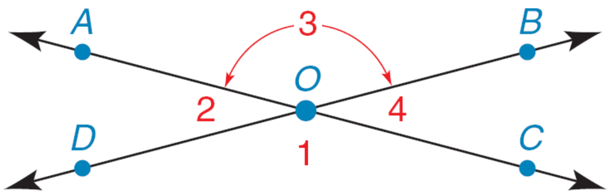

Figure 1.62

Reflexive: $\angle 1 \cong \angle 1$; an angle is congruent to itself.

Symmetric: If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive: If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Relations

Any relation (such as congruence of angles) that has reflexive, symmetric, and transitive properties is known as an *equivalence relation*.

Relations

Theorem 1.6.2

If two lines intersect, then the vertical angles formed are congruent.

CONSTRUCTIONS LEADING TO PERPENDICULAR LINES

Constructions Leading to Perpendicular Lines

Construction 5

To construct the line perpendicular to a given line at a specified point on the given line.

Given: $\overleftrightarrow{AB}$ with point *X* in Figure 1.63(a)

Figure 1.63(a)

Construct: A line $\overleftrightarrow{EX}$, so that $\overleftrightarrow{EX} \perp \overleftrightarrow{AB}$

Constructions Leading to Perpendicular Lines

Construction: Figure 1.63(b): Using X as the center, mark off arcs of equal radii on each side of X to intersect $\overleftrightarrow{AB}$ at C and D.

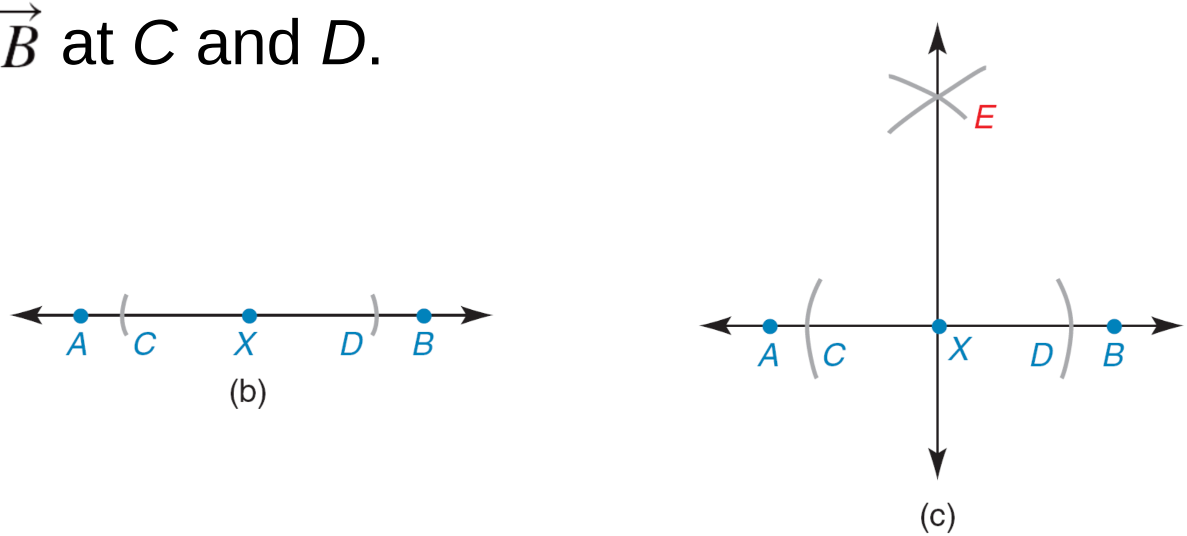

Figure 1.63

Figure 1.63(c): Now, using C and D as centers, mark off arcs of equal radii with a length greater than XD so that these arcs intersect either above (as shown) or below $\overleftrightarrow{AB}$.

Constructions Leading to Perpendicular Lines

Calling the point of intersection *E*, draw $\overleftrightarrow{EX}$, which is the desired line; that is, $\overleftrightarrow{EX} \perp \overleftrightarrow{AB}$.

Theorem 1.6.3

In a plane, there is exactly one line perpendicular to a given line at any point on the line.

Theorem 1.6.4

The perpendicular bisector of a line segment is unique.