Chapter 1

Line and Angle Relationships

1.5 Introduction to Geometric Proof

Introduction to Geometric Proof

We use the Addition Property of Equality to justify adding the same number to each side of an equation. Reasons found in a proof often include the properties found in Tables 1.5 and 1.6.

TABLE 1.5

Properties of Equality (a, b, and c are real numbers)

Addition Property of Equality:	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality:	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality:	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality:	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Introduction to Geometric Proof

TABLE 1.6

Further Algebraic Properties of Equality (a, b, and c are real numbers)

Reflexive Property:	$a = a$.
Symmetric Property:	If $a = b$, then $b = a$.
Distributive Property:	$a(b + c) = a \cdot b + a \cdot c$.
Substitution Property:	If $a = b$, then a replaces b in any equation.
Transitive Property:	If $a = b$ and $b = c$, then $a = c$.

As we discover in Example 1, some properties can be used interchangeably.

Example 1

Which property of equality justifies each conclusion?

a) If $2x - 3 = 7$, then $2x = 10$. b) If $2x = 10$, then $x = 5$.

Solution:

a) Addition Property of Equality; added 3 to each side of the equation.

b) Multiplication Property of Equality; multiplied each side of the equation by $\frac{1}{2}$. *OR* Division Property of Equality; divided each side of the equation by 2.

Introduction to Geometric Proof

Some properties of inequality (see Table 1.7) are useful in geometric proof.

TABLE 1.7

Properties of Inequality (a, b, and c are real numbers)

Addition Property of Inequality:	If $a > b$, then $a + c > b + c$.
	If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality:	If $a > b$, then $a - c > b - c$.
	If $a < b$, then $a - c < b - c$.

SAMPLE PROOFS

Sample Proofs

Consider Figure 1.56 and this problem:

Given: $MN > PQ$
Prove: $MP > NQ$

Figure 1.56

To understand the situation, first study the Drawing (Figure 1.56) and the related Given. Then read the Prove with reference to the Drawing.

What may be confusing here is that the Given involves *MN* and *PQ*, whereas the Prove involves *MP* and *NQ*.

Sample Proofs

However, this is easily remedied through the addition of NP to each side of the inequality $MN > PQ$.

Example 5

Given : $MN > PQ$ (Figure 1.57)

Prove : $MP > NQ$

Figure 1.57

Proof:

Statements	**Reasons**
1. $MN > PQ$	1. Given
2. $MN + NP > NP + PQ$	2. Addition Property of Inequality
3. $MN + NP = MP$ and $NP + PQ = NQ$	3. Segment-Addition Postulate
4. $MP > NQ$	4. Substitution