Chapter 1

Line and Angle Relationships

1.2 Informal Geometry and Measurement

Informal Geometry and Measurement

In geometry, the terms *point, line,* and *plane* are described but not defined.

Other concepts that are accepted intuitively, but never defined, include the *straightness* of a line, the *flatness* of a plane, the notion that a point on a line lies *between* two other points on the line, and the notion that a point lies in the *interior* or *exterior* of an angle.

Informal Geometry and Measurement

The following are descriptions of some of the undefined terms.

A **point**, which is represented by a dot, has location but not size; that is, a point has no dimensions. An uppercase italic letter is used to name a point.

Figure 1.8 shows points *A*, *B*, and *C*.

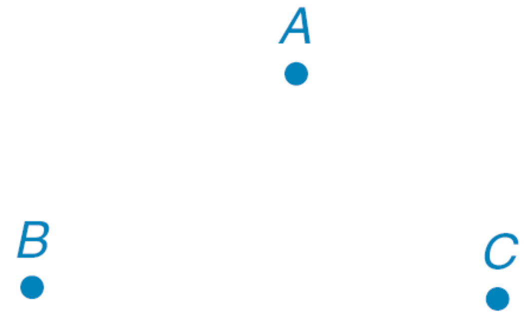


Figure 1.8

Informal Geometry and Measurement

The second undefined geometric term is **line**. A line is an infinite set of points. Given any two points on a line, there is always a point that lies between them on that line.

Lines have a quality of “straightness” that is not defined but assumed. Given several points on a line, these points form a straight path.

Whereas a point has no dimensions, a line is one-dimensional; that is, the distance between any two points on a given line can be measured.

Informal Geometry and Measurement

Line *AB*, represented symbolically by $\overleftrightarrow{AB}$, extends infinitely far in opposite directions, as suggested by the arrows on the line. A line may also be represented by a single lowercase letter.

Figures 1.9(a) and (b) show the lines *AB* and *m*.

When a lowercase letter is used to name a line, the line symbol is omitted; that is, $\overleftrightarrow{AB}$ and *m* can name the same line.

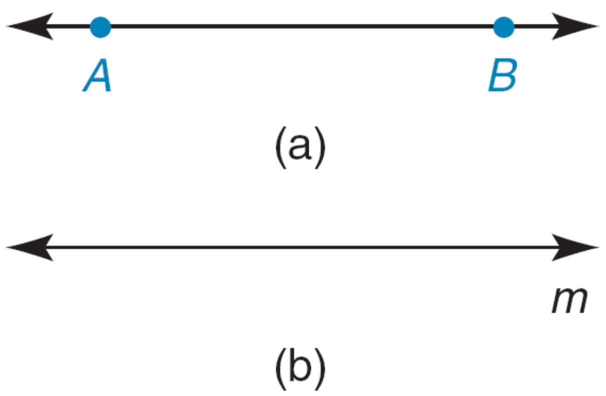


Figure 1.9

Informal Geometry and Measurement

Note the position of point *X* on $\overleftrightarrow{AB}$ in Figure 1.9(c).

Figure 1.9 (c)

When three points such as *A*, *X*, and *B* are on the same line, they are said to be **collinear**.

In the order shown, which is symbolized *A-X-B* or *B-X-A*, point *X* is said to be *between* *A* and *B*.

Informal Geometry and Measurement

When a drawing is not provided, the notation *A*-*B*-*C* means that these points are collinear, with *B between A* and *C*.

When a drawing is provided, we assume that all points in the drawing that appear to be collinear *are* collinear, *unless otherwise stated*.

Figure 1.9(d) shows that *A*, *B*, and *C* are collinear, in Figure 1.8, points *A*, *B*, and *C* are *noncollinear*.

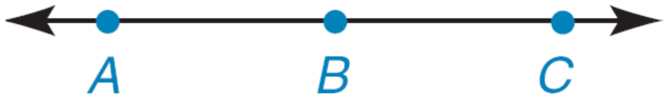


Figure 1.9 (d)

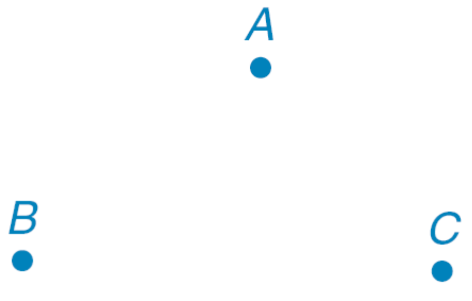


Figure 1.8

Informal Geometry and Measurement

At this time, we informally introduce some terms that will be formally defined later.

You have probably encountered the terms *angle*, *triangle*, and *rectangle* many times. An example of each is shown in Figure 1.10.

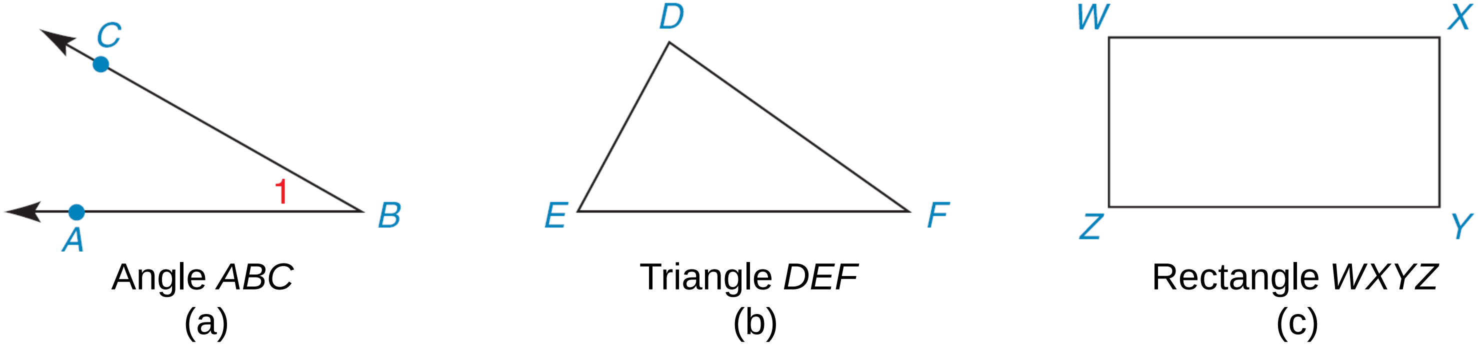
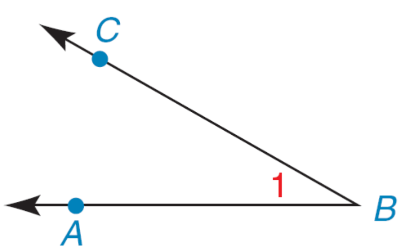


Figure 1.10

Informal Geometry and Measurement

Using symbols, we refer to Figures 1.10(a), (b), and (c) as $\angle ABC$, $\triangle DEF$, and $\square$ *WXYZ*, respectively.

Some caution must be used in naming figures; although the angle in Figure 1.10(a) can be called $\angle CBA$, it is incorrect to describe the angle as $\angle ACB$ because that order implies a path from point *A* to point *C* to point *B* . . . a different angle!



Angle *ABC*

(a)

Figure 1.10

Informal Geometry and Measurement

In $\angle ABC$, the point *B* at which the sides meet is called the **vertex** of the angle. Because there is no confusion regarding the angle described, $\angle ABC$ is also known as $\angle B$ (using only the vertex) or as $\angle 1$.

The points *D*, *E*, and *F* at which the sides of $\triangle DEF$ (also called $\triangle DFE$, $\triangle EFD$, etc.) meet are called the *vertices* (plural of *vertex*) of the triangle.

Similarly, *W*, *X*, *Y*, and *Z* are the vertices of the rectangle; the vertices are named in an order that traces the rectangle.

Informal Geometry and Measurement

A **line segment** is part of a line. It consists of two distinct points on the line and all points between them.
(See Figure 1.11.)

Using symbols, we indicate the line segment by $\overline{BC}$; note that $\overline{BC}$ is a set of points but is not a number.

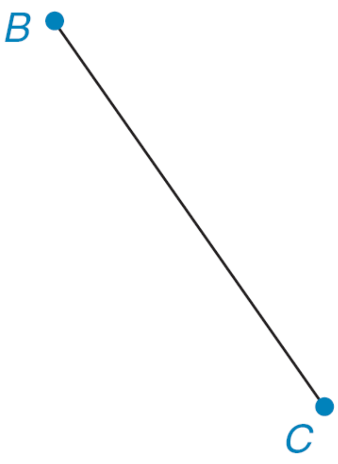


Figure 1.11

We use *BC* (omitting the segment symbol) to indicate the *length* of this line segment; thus, *BC* is a number. The sides of a triangle or rectangle are line segments.

Example 1

Can the rectangle in Figure 1.10(c) be named

a) ▭ *XYZW*?

b) ▭ *WYXZ*?

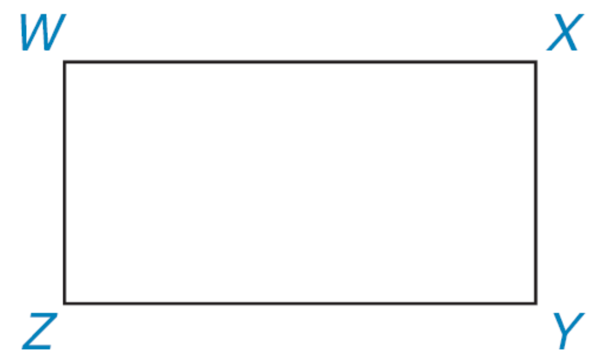


Figure 1.10 (c)

Solution:

a) Yes, because the points taken in this order trace the figure.

b) No; for example, $\overline{WY}$ is not a side of the rectangle.

MEASURING LINE SEGMENTS

Measuring Line Segments

The instrument used to measure a line segment is a scaled straightedge such as a *ruler*, a *yardstick*, or a *meter stick*.

Line segment *RS* ($\overline{RS}$ in symbols) in Figure 1.12 measures 5 centimeters. Because we express the length of $\overline{RS}$ by *RS* (with no bar), we write *RS* = 5 cm.

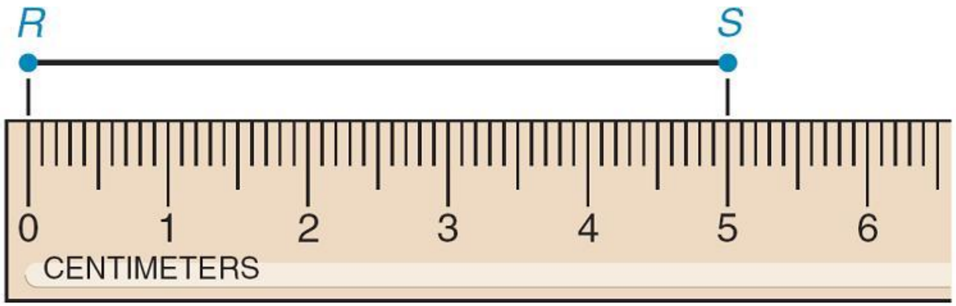


Figure 1.12

Measuring Line Segments

To find the length of a line segment using a ruler:

1. Place the ruler so that “0” corresponds to one endpoint of the line segment.

2. Read the length of the line segment by reading the number at the remaining endpoint of the line segment.

Because manufactured measuring devices such as the ruler, yardstick, and meter stick may lack perfection or be misread, there is a margin of error each time one is used.

Measuring Line Segments

In Figure 1.12, for instance, *RS* may actually measure 5.02 cm (and that could be rounded from 5.023 cm, etc.). Measurements are approximate, not perfect.

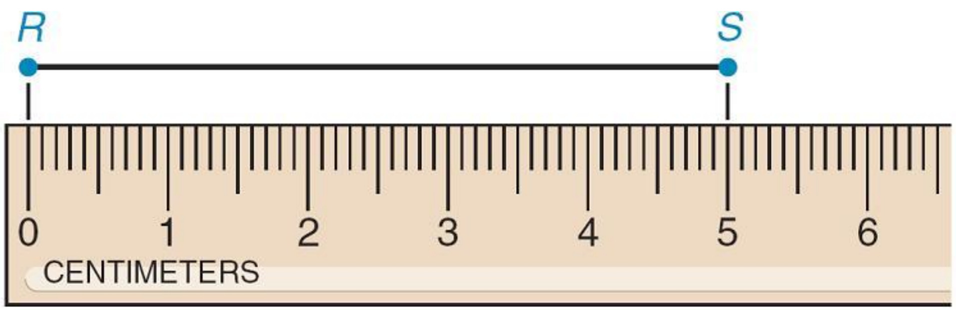


Figure 1.12

Measuring Line Segments

In Example 2, a ruler (not drawn to scale) is shown in Figure 1.13. In the drawing, the distance between consecutive marks on the ruler corresponds to 1 inch. The measure of a line segment is known as *linear measure*.

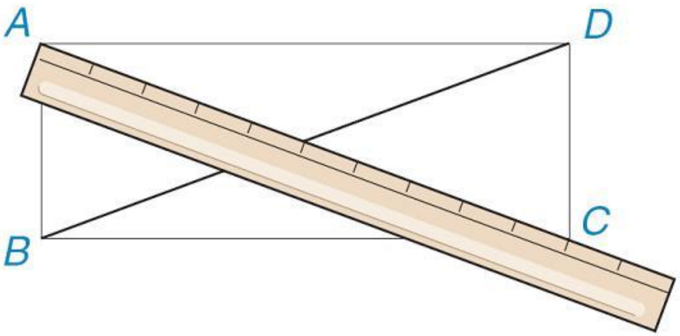


Figure 1.13

Example 2

In rectangle *ABCD* of Figure 1.13, the line segments $\overline{AC}$ and $\overline{BD}$ shown are the diagonals of the rectangle. How do the lengths of the diagonals compare?

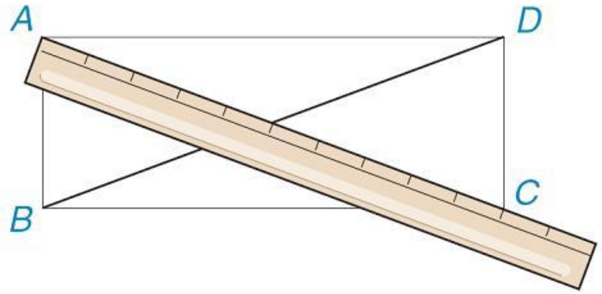


Figure 1.13

Solution:

As shown on the ruler, *AC* = 10”. As intuition suggests, the lengths of the diagonals are the same, so it follows that *BD* = 10”.

Note: In linear measure, 10” means 10 inches, and 10’ means 10 feet.

Measuring Line Segments

In Figure 1.14, point *B* lies between *A* and *C* on $\overline{AC}$. If $AB = BC$, then *B* is the **midpoint** of $\overline{AC}$.

Figure 1.14

When $AB = BC$, the geometric figures $\overline{AB}$ and $\overline{BC}$ are said to be **congruent.** Numerical lengths may be equal, but the actual line segments (geometric figures) are congruent.

The symbol for congruence is ≅; thus, $\overline{AB} \cong \overline{BC}$ if *B* is the midpoint of $\overline{AC}$.

MEASURING ANGLES

Measuring Angles

An angle's measure depends not on the lengths of its sides but on the amount of opening between its sides.

In Figure 1.16, the arrows on the angles' sides suggest that the sides extend indefinitely.

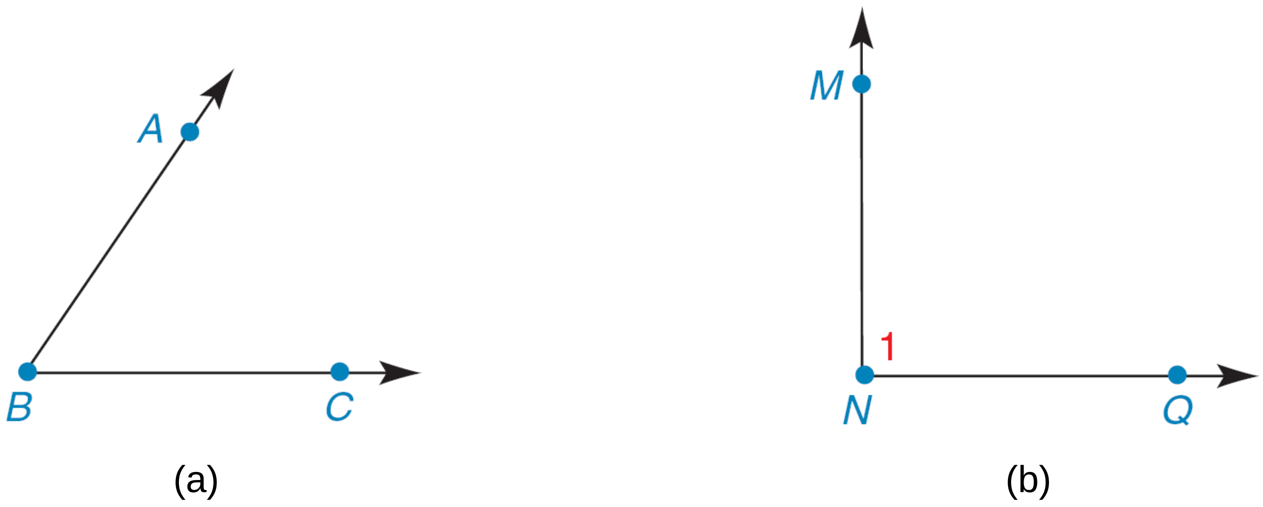


Figure 1.16

Measuring Angles

The instrument shown in Figure 1.17 (and used in the measurement of angles) is a **protractor**.

For example, you would express the measure of $\angle RST$ by writing $m\angle RST = 50°$; this statement is read, "The measure of $\angle RST$ is 50 degrees."

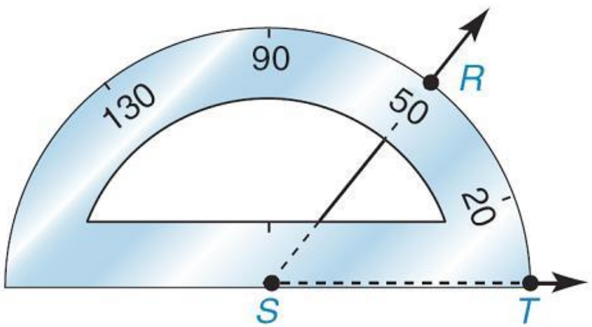


Figure 1.17

Measuring the angles in Figure 1.16 with a protractor, we find that $m\angle B = 55°$ and $m\angle 1 = 90°$.

Measuring Angles

If the degree symbol is missing, the measure is understood to be in degrees; thus $m\angle 1 = 90$.

In practice, the protractor shown will measure an angle that is greater than 0° but less than or equal to 180°.

Measuring Angles

To find the degree measure of an angle using a protractor:

1. Place the notch of the protractor at the point where the sides of the angle meet (the vertex of the angle). See point *S* in Figure 1.18.

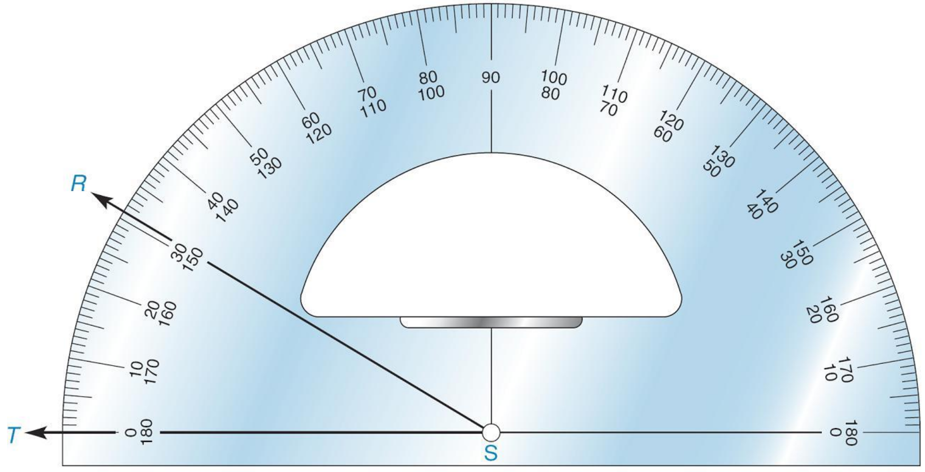


Figure 1.18

Measuring Angles

2. Place the edge of the protractor along a side of the angle so that the scale reads “0.” See point *T* in Figure 1.18 where we use “0” on the outer scale.

3. Using the same (outer) scale, read the angle size by reading the degree measure that corresponds to the second side of the angle.

Example 4

For Figure 1.18, find the measure of $\angle RST$.

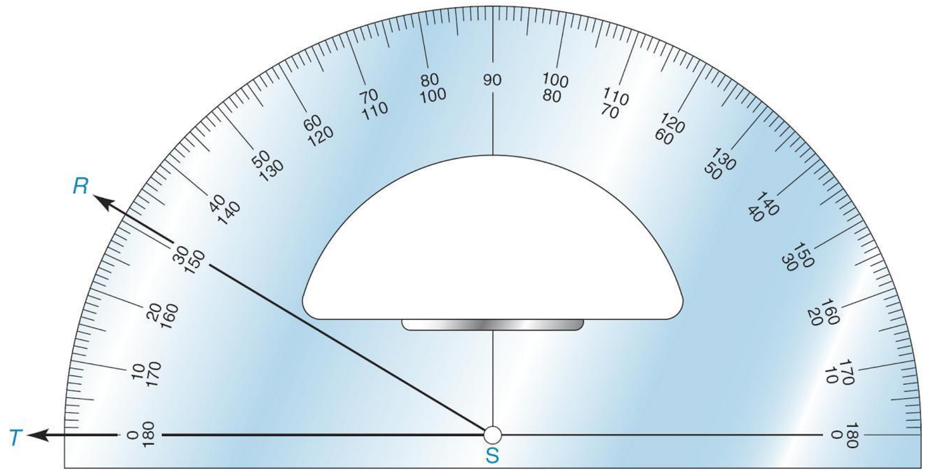


Figure 1.18

Solution:

Using the protractor, we find that the measure of angle *RST* is 31°. (In symbols, $m\angle RST = 31°$ or $m\angle RST = 31$.)

Measuring Angles

Some protractors show a full 360°; such a protractor is used to measure an angle whose measure is between 0° and 360°. An angle whose measure is between 180° and 360° is known as a *reflex angle*.

Just as measurement with a ruler is not perfect, neither is measurement with a protractor.

Measuring Angles

The lines on a sheet of paper in a notebook are *parallel*.

Informally, **parallel** lines lie on the same page and will not cross over each other even if they are extended indefinitely.

We say that lines ℓ and m in Figure 1.19(a) are parallel; note here the use of a lowercase letter to name a line.

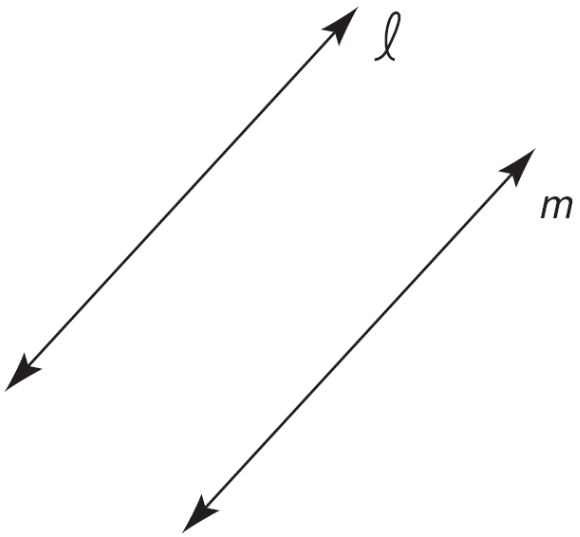


Figure 1.19 (a)

Measuring Angles

We say that line segments are parallel if they are parts of parallel lines; if $\overleftrightarrow{RS}$ is parallel to $\overleftrightarrow{MN}$, then $\overline{RS}$ is parallel to $\overline{MN}$ in Figure 1.19(b).

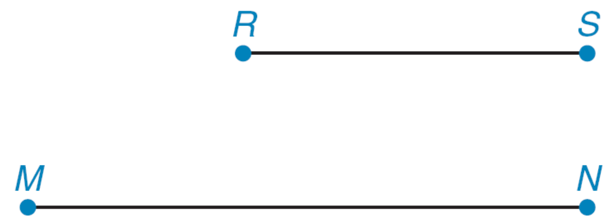


Figure 1.19 (b)

For $A = \{1, 2, 3\}$ and $B = \{6, 8, 10\}$, there are no common elements; for this reason, we say that the intersection of A and B is the **empty set** (symbol is $\emptyset$). Just as $A \cap B = \emptyset$, the parallel lines in Figure 1.19(a) are characterized by $\ell \cap m = \emptyset$.

Example 5

In Figure 1.20 the sides of angles *ABC* and *DEF* are parallel ($\overline{AB}$ to $\overline{DE}$ and $\overline{BC}$ to $\overline{EF}$). Use a protractor to decide whether these angles have equal measures.

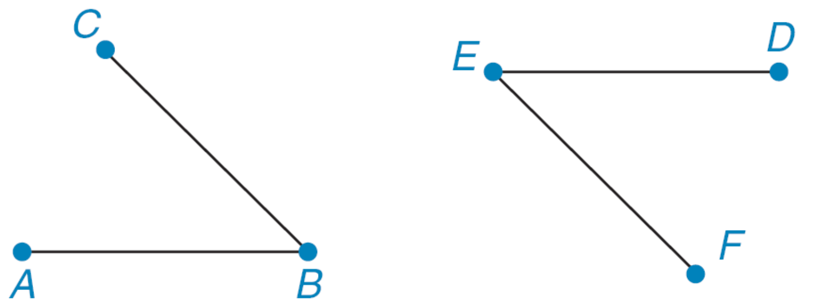


Figure 1.20

Solution:

The angles have equal measures. Both measure 44°.

Measuring Angles

Two angles with equal measures are said to be *congruent.*
In Figure 1.20, we see that $\angle ABC \cong \angle DEF$.
In Figure 1.21, $\angle ABC \cong \angle CBD$.

In Figure 1.21, angle *ABD* has been separated into smaller angles *ABC* and *CBD;* if the two smaller angles are congruent (have equal measures), then angle *ABD* has been *bisected*.

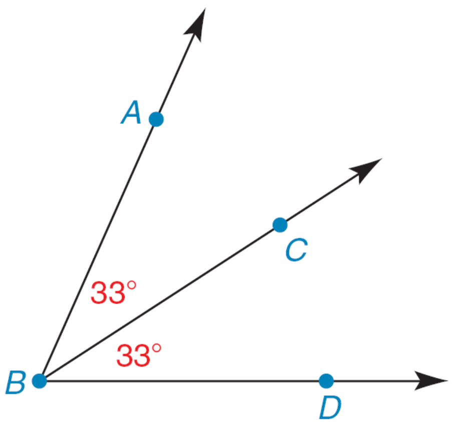


Figure 1.21

In general, the word **bisect** means to separate a line segment (or an angle) into two parts of equal measure.

Measuring Angles

Any angle having a 180° measure is called a **straight angle**, an angle whose sides are in opposite directions. See straight angle *RST* in Figure 1.22(a).

When a straight angle is bisected, as shown in Figure 1.22(b), the two angles formed are **right angles** (each measures 90°).

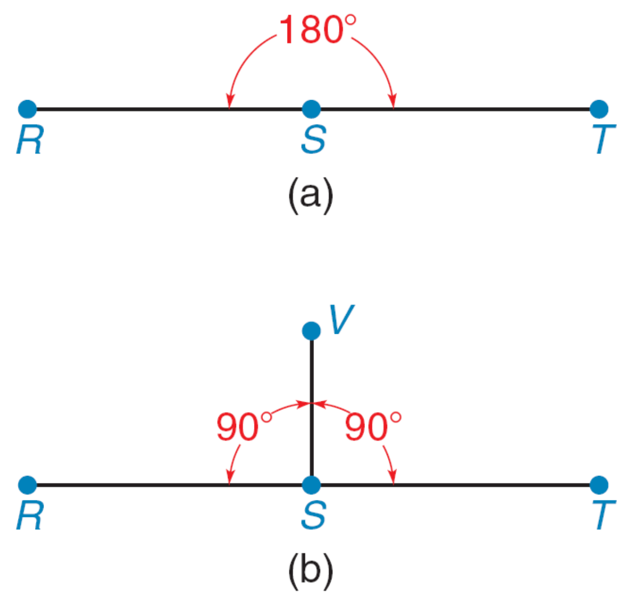


Figure 1.22

Measuring Angles

When two lines have a point in common, as in Figure 1.23, they are said to **intersect**.

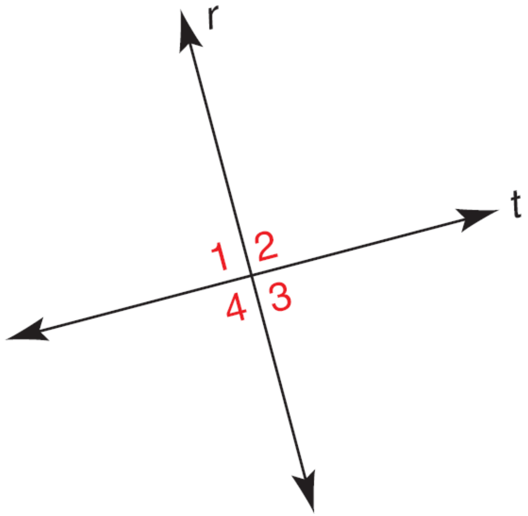


Figure 1.23

When two lines intersect and form congruent adjacent angles, they are said to be **perpendicular**.

CONSTRUCTIONS

Constructions

Another tool used in geometry is the **compass**. This instrument, shown in Figure 1.24, is used to draw circles and parts of circles known as *arcs*.

The ancient Greeks insisted that only two tools (a compass and a straightedge) be used for geometric **constructions**, which were idealized drawings assuming perfection in the use of these tools.

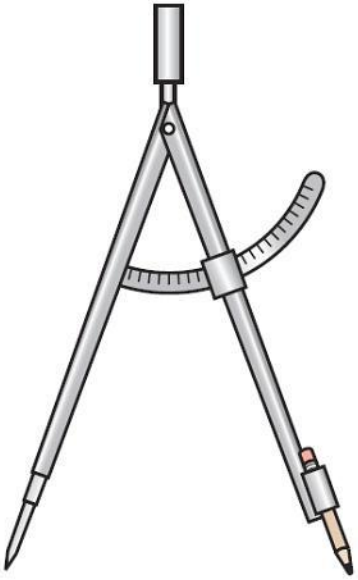


Figure 1.24

Constructions

The compass was used to create “perfect” circles and for marking off segments of “equal” length.

The straightedge could be used to draw a straight line through two designated points.

A **circle** is the set of all points in a plane that are at a given distance from a particular point (known as the “center” of the circle).

The part of a circle between any two of its points is known as an **arc**.

Constructions

Any line segment joining the center to a point on the circle is a **radius** (plural: *radii*) of the circle. See Figure 1.25.

Construction 1, which follows, is quite basic and depends only on using arcs of the same radius length to construct line segments of the same length.

The arcs are created by using a compass.

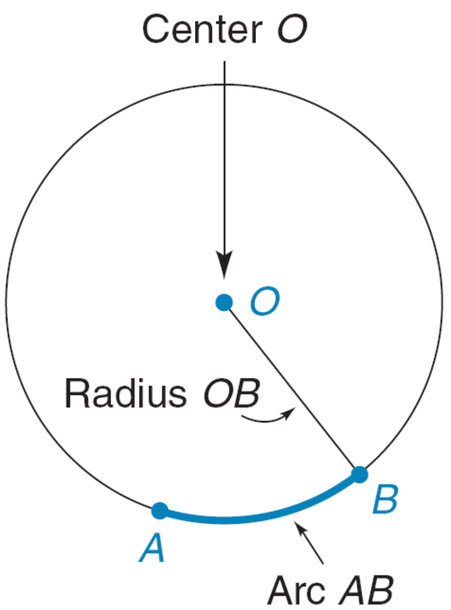


Figure 1.25

Constructions

Construction 1

To construct a segment congruent to a given segment.

Given: $\overline{AB}$ in Figure 1.26(a).

Figure 1.26 (a)

Construct: $\overline{CD}$ on line *m* so that $\overline{CD} \cong \overline{AB}$ (or $CD = AB$)

Constructions

Construction:

With your compass open to the length of $\overline{AB}$, place the stationary point of the compass at *C* and mark off a length equal to *AB* at point *D*, as shown in Figure 1.26(b).

Then *CD* = *AB*.

Figure 1.26 (b)

Constructions

Construction 2

To construct the midpoint *M* of a given line segment *AB*.

Given: $\overline{AB}$ in Figure 1.27(a).

Construct: *M* on $\overline{AB}$ so that *AM* = *MB*

Figure 1.27 (a)

Constructions

Construction:

Figure 1.27(a): Open your compass to a length greater than one-half of $\overline{AB}$.

Figure 1.27(b): Using *A* as the center of the arc, mark off an arc that extends both above and below segment *AB*.

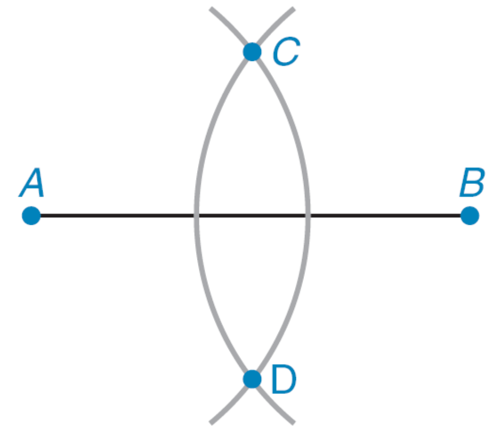


Figure 1.27 (b)

Constructions

With *B* as the center and keeping the same length of radius, mark off an arc that extends above and below $\overline{AB}$ so that two points (*C* and *D*) are determined where the arcs cross.

Figure 1.27(c): Now draw $\overline{CD}$.

The point where $\overline{CD}$ crosses $\overline{AB}$ is the midpoint *M*.

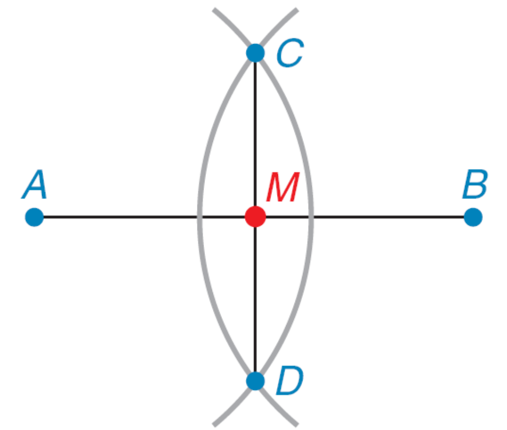


Figure 1.27 (c)

Example 7

In Figure 1.28, *M* is the midpoint of $\overline{AB}$.

a) Find *AM* if *AB* = 15.

b) Find *AB* if *AM* = 4.3.

c) Find *AB* if *AM* = $2x + 1$.

Figure 1.28

Solution:

a) *AM* is one-half of *AB*, so *AM* = $7\frac{1}{2}$.

b) *AB* is twice *AM*, so *AB* = 2(4.3) or *AB* = 8.6.

c) *AB* is twice *AM*, so *AB* = $2(2x + 1)$ or *AB* = $4x + 2$.