



# Fractions

Prepared by Johnny Howard © 2015 South-Western, a part of Cengage Learning

#### **Learning Objectives**

By studying this chapter and completing all assignments, you will be able to:



Figure 1-1 a. 1 whole pizza  $=\frac{6}{6}$  b. 2 pieces  $=\frac{2}{6}$ c. 2 whole pizzas  $=\frac{12}{6}$ 

d. 1 piece missing  $=\frac{11}{6} = 1\frac{5}{6}$ 

#### **STEPS** to Change an Improper Fraction to a Mixed Number

- 1. Divide the numerator by the denominator.
- 2. The quotient is the whole-number part of the mixed number.
- 3. The remainder is the numerator of the fraction part.
- 4. The original denominator is the denominator of the fraction part.

#### **EXAMPLE** A

## Change $\frac{11}{8}$ to a mixed number.



*Note:* Refer to Point A in Figure 1-2 to see where these numbers appear on a ruler.

Figure 1-2



#### **STEPS** to Change a Mixed Number to an Improper Fraction

- 1. Multiply the denominator of the fraction part by the whole number.
- 2. Add the numerator of the fraction part to the product of Step 1. The sum is the numerator of the improper fraction.
- 3. The denominator of the fraction part of the mixed number is the denominator of the improper fraction.

#### Example B



*Note:* Refer to Point B in Figure 1-2 to see where these numbers appear on a ruler.

#### **STEPS** to Reduce a Fraction to Lowest Terms

- Divide both the numerator and the denominator by a common divisor greater than 1 to arrive at a reduced fraction.
- 2. If necessary, repeat Step 1 until the fraction cannot be reduced any further.
- *Note:* If a fraction's numerator and denominator have no common divisor greater than 1, the fraction is already in lowest terms.

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Learning Objective

#### EXAMPLE C

Reduce 
$$\frac{12}{16}$$
 to lowest terms.  
 $\frac{12}{16} = \frac{12 \div 2}{16 \div 2} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ 
or else  $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$ 

*Note:* Dividing by 4 once is faster than dividing by 2 twice. Always use the greatest common divisor that you can find.

#### **STEPS** to Raise a Fraction to Higher Terms

- 1. Divide the new denominator by the old denominator. The quotient is the *common multiplier*.
- 2. Multiply the old numerator by the common multiplier.
- 3. Multiply the old denominator by the common multiplier.

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Learning Objective

#### **EXAMPLE D**

## Raise $\frac{3}{4}$ to twenty-fourths.



#### **STEPS** to Add Two or More Fractions and/or Mixed Numbers

- 1. If necessary, change the fraction parts to fractions with common denominators. The common denominator is the denominator in the fraction part of the answer.
- 2. Add the numerators to make the numerator of the fraction part of the answer. If there are any whole-number parts, add them to make the whole-number part of the answer.
- 3. If necessary, change an improper fraction to a mixed number and mentally add any whole number parts.
- 4. Reduce the fraction part of the answer to lowest terms.

#### EXAMPLE E

Add  $2\frac{7}{8}$  and  $4\frac{3}{8}$ . The fractions already have a common denominator of 8.  $2\frac{7}{8}$  $+4\frac{3}{-}$  $6\frac{10}{8} = 6 + 1\frac{2}{8} = 7\frac{1}{4}$ STEP 2 STEP 3 STEP 4

#### EXAMPLE F

Add  $\frac{5}{6}$  and  $\frac{3}{4}$ . A common denominator is  $6 \times 4 = 24$ .

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$
  
+  $\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$   
$$\frac{38}{24} = 1\frac{14}{24} = 1\frac{7}{12}$$
  
STEPS 2, 3, & 4

#### EXAMPLE G

Add  $3\frac{5}{8}$  and  $7\frac{5}{6}$ . The least common denominator is 24.



#### Figure 1-3



a. 3 whole units plus  $\frac{1}{4}$  of a unit



b. 2 "whole units" plus  $\frac{5}{4}$  of a unit

## **STEPS** to Subtract One Fraction or Mixed Number from Another

- 1. If necessary, change the fractions so that all fractions have a common denominator. The common denominator is the denominator in the fraction part of the answer.
- 2. If necessary, "borrow 1" from the whole-number part of the minuend so that the fraction part of the minuend is at least as large as the fraction part of the subtrahend.
- 3. Subtract the numerators in the fractions to make the numerator in the fraction part of the answer.
- 4. If there are any whole-number parts, subtract them to make the whole-number part of the answer.
- 5. Reduce the fraction part of the answer to lowest terms.

#### **EXAMPLES** H and I





#### EXAMPLES J and K





#### **STEPS** to Multiply Fractions and Mixed Numbers

- 1. Change any mixed (or whole) numbers to improper fractions.
- 2. Multiply all the numerators to get the numerator of the product.
- 3. Multiply all the denominators to get the denominator of the product.
- 4. Change the product to a proper fraction or mixed number in lowest terms.

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Learning Objective

#### EXAMPLES L and M



#### **EXAMPLES N, O and P**



#### **STEPS** to Divide Fractions and Mixed Numbers

- 1. Change any mixed or whole numbers to improper fractions.
- 2. Invert the divisor (that is, exchange the numerator and denominator).
- 3. Change the division symbol to a multiplication symbol.
- 4. Multiply the two factors (canceling where possible, if desired).
- 5. Write the result as a proper fraction or mixed number in lowest terms.

Learning Objective

#### **EXAMPLES Q** and **R**

STEPS 2 & 3  

$$\frac{3}{10} \div \frac{2}{5} = \frac{3}{10} \times \frac{5}{2} = \frac{3}{10} \times \frac{5}{2} = \frac{3}{10} \times \frac{3}{2} = \frac{3 \times 1}{2 \times 2} = \frac{3}{4}$$

STEP 1
 STEPS 2 & 3
 STEP 4
 STEP 5

 
$$6 \div 1\frac{3}{5} = \frac{6}{1} \div \frac{8}{5} = \frac{6}{1} \times \frac{5}{8} = \frac{\frac{3}{6}}{1} \times \frac{5}{8} = \frac{3 \times 5}{1 \times 4} = \frac{15}{4} = 3\frac{3}{4}$$
 $3\frac{3}{4}$ 

### Chapter Terms for Review

cancel cancellation common denominator denominator fractions higher termslowest termsimproper fractionmixed numberleast common denominatornumeratorlower termsproper fraction

A Change the improper fractions to whole numbers or to mixed numbers. Change the mixed numbers to improper fractions.



In problems 13–20, reduce each fraction to lowest terms. In problems 21–27, raise each fraction to higher terms, as indicated.



Learning Objectives

**B**)

C Add the following fractions and mixed numbers. Write the answers as fractions or mixed numbers, with fractions in lowest terms.

D Subtract the following fractions and mixed numbers. Write the answers as proper fractions or mixed numbers, with fractions in lowest terms.

$$36. \quad \frac{5}{8} \\ -\frac{3}{8} \\ -\frac{3}{8} \\ \frac{2}{8} = \frac{1}{4} \\ 40. \quad 3\frac{2}{3} = 3\frac{4}{6} = 2\frac{10}{6} \\ -\frac{13}{6} \\ -\frac{13}{6} \\ -\frac{12}{12} = 1\frac{1}{6} \\ \frac{12}{12} = 1\frac{1}{6} \\ -\frac{12}{12} = 1\frac{1}{6} \\ \frac{12}{12} = 1\frac{1}{6} \\ \frac{9}{35} \\ -\frac{11}{12} = -1\frac{1}{12} \\ \frac{9}{35} \\ -\frac{11}{12} = -1\frac{1}{12} \\ \frac{9}{35} \\ \frac{18}{12} = 1\frac{2}{3} \\ \frac{9}{35} \\ \frac{18}{12} = 1\frac{2}{3} \\ \frac{9}{35} \\ \frac{18}{12} = 1\frac{2}{3} \\ 41. \quad 3\frac{3}{5} = 3\frac{12}{20} = 2\frac{32}{20} \\ -\frac{3}{20} = 2\frac{32}{20} \\ -\frac{3}{20} \\ -\frac{3}{45} \\ -\frac{4}{9} = 4\frac{20}{45} \\ -\frac{15}{6} = -1\frac{25}{30} = -1\frac{25}{30} \\ -\frac{15}{6} = -1\frac{25}{30} = -1\frac{25}{30} \\ -\frac{15}{6} = -1\frac{25}{30} = -1\frac{25}{30} \\ -\frac{17}{30} \\ -\frac{11}{30} \\$$

- **E** Business Applications and Critical Thinking. Solve the following. Write your answers as fractions or mixed numbers in lowest terms.
- **44.** A restaurant sells three different hamburgers, based on the amount of meat used: "The Mini"  $(\frac{1}{4} \text{ lb})$ ; "The Regular"  $(\frac{1}{3} \text{ lb})$ ; and "The Maxi"  $(\frac{1}{2} \text{ lb})$ . Students bought one of each to compare them. What was the total amount of meat used in the three hamburgers?  $1\frac{1}{12}$  lb

 $\frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12} = 1\frac{1}{12}$  lb

45. Lynn Atkins specializes in custom faux painting, but for the first coat he could combine leftover paints when the colors were relatively the same. He has three containers of different shades of white: 2<sup>2</sup>/<sub>3</sub> quarts, 3<sup>1</sup>/<sub>4</sub> quarts, and 2<sup>1</sup>/<sub>2</sub> quarts. If Lynn combines the contents of all the containers, how many quarts of paint will he have? 8<sup>5</sup>/<sub>12</sub> qt

 $2\frac{2}{3} + 3\frac{1}{4} + 2\frac{1}{2} = 2\frac{8}{12} + 3\frac{3}{12} + 2\frac{6}{12} = 7\frac{17}{12} = 8\frac{5}{12}qt$ 

**46.** Contractor Dick Reiling has a top board that is  $\frac{11}{16}$  inch thick. Dick wants to use wood screws to attach it to a bottom board. If a wood screw is  $1\frac{1}{4}$  inches long, how much of the screw will be left over to go into the bottom board?  $\frac{9}{16}$  in.

 $1\frac{1}{4} - \frac{11}{16} = 1\frac{4}{16} - \frac{11}{16} = \frac{20}{16} - \frac{11}{16} = \frac{9}{16}$  in.

- **E** Business Applications and Critical Thinking. Solve the following. Write your answers as fractions or mixed numbers in lowest terms. (cont'd)
- 47. Robert Landles is planning to attach a plywood panel to a wall using nails that are 1<sup>3</sup>/<sub>4</sub> inches long. The panel is <sup>3</sup>/<sub>8</sub> inch thick. Beneath the panel is a layer of sheetrock that is <sup>1</sup>/<sub>2</sub> inch thick. How many inches of the nail should go into the wood frame that is underneath the sheetrock? <sup>7</sup>/<sub>8</sub> in.
  <sup>3</sup>/<sub>8</sub> + <sup>1</sup>/<sub>2</sub> = <sup>3</sup>/<sub>8</sub> + <sup>4</sup>/<sub>8</sub> = <sup>7</sup>/<sub>8</sub> in. (plywood plus sheetrock) 1<sup>3</sup>/<sub>4</sub> <sup>7</sup>/<sub>8</sub> = 1<sup>6</sup>/<sub>8</sub> <sup>7</sup>/<sub>8</sub> = <sup>14</sup>/<sub>8</sub> <sup>7</sup>/<sub>8</sub> = <sup>7</sup>/<sub>8</sub> in. to go into wood frame
- **48.** Milan Fabric Center sold four pieces of wool fabric to a tailor. The pieces measured  $3\frac{1}{4}$  yards,  $2\frac{1}{3}$  yards,  $1\frac{3}{4}$  yards, and  $4\frac{1}{2}$  yards in length. How many yards of fabric did the tailor purchase?  $11\frac{5}{6}$  yd  $3\frac{1}{4} + 2\frac{1}{3} + 1\frac{3}{4} + 4\frac{1}{2} = 3\frac{3}{12} + 2\frac{4}{12} + 1\frac{9}{12} + 4\frac{6}{12} = 10\frac{22}{12} = 11\frac{10}{12} = 11\frac{5}{6}$  yd purchased

Change whole or mixed numbers to improper fractions and multiply. Cancel if possible. Where the word of appears, replace it with the multiplication symbol. Write the answers as mixed numbers or proper fractions in lowest terms.



A

B Change the mixed numbers to improper fractions and divide. Cancel where possible. Write the quotients as mixed numbers or proper fractions in lowest terms.

$$9. \frac{7}{8} \div \frac{3}{4} = \frac{1\frac{1}{6}}{\frac{7}{8} \div \frac{3}{4} = \frac{7 \times 1}{8} \times \frac{4}{3} = \frac{7 \times 1}{2 \times 3} = \frac{7}{6} = 1\frac{1}{6}$$

$$10. \frac{3}{10} \div \frac{9}{20} = \frac{2}{3}$$

$$\frac{3}{10} \div \frac{9}{20} = \frac{3}{10} \times \frac{20}{9} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$$

$$11. \frac{5}{6} \div \frac{4}{9} = \frac{1\frac{7}{8}}{\frac{5}{6} \div \frac{9}{9} = \frac{5 \times 3}{2 \times 4} = \frac{15}{8} = 1\frac{7}{8}$$

$$12. \frac{7}{10} \div 2\frac{4}{5} = \frac{1}{4}$$

$$\frac{7}{10} \div \frac{14}{5} = \frac{7}{10} \times \frac{5}{14} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

$$13. 6\frac{1}{4} \div 4\frac{3}{8} = \frac{1\frac{3}{7}}{\frac{7}{10} \times \frac{8}{5} = \frac{5 \times 2}{1 \times 7} = \frac{10}{7} = 1\frac{3}{7}$$

$$14. 1\frac{7}{8} \div 5 = \frac{3}{8}$$

$$\frac{15}{8} \div \frac{5}{1} = \frac{15}{8} \times \frac{1}{5} = \frac{3 \times 1}{8 \times 1} = \frac{3}{8}$$

$$15. 3\frac{1}{3} \div \frac{4}{5} = \frac{4\frac{1}{6}}{\frac{10}{3} \times \frac{5}{4} = \frac{5 \times 5}{3 \times 2} = \frac{25}{6} = 4\frac{1}{6}$$

$$16. 4\frac{1}{6} \div 3\frac{3}{4} = \frac{1\frac{1}{9}}{\frac{25}{6} \times \frac{4}{15} = \frac{5 \times 2}{3 \times 3} = \frac{10}{9} = 1\frac{1}{9}$$

- C Business Applications and Critical Thinking. Use fractions and mixed numbers to solve each of the following. State the answers as whole numbers, proper fractions, or mixed numbers in lowest terms.
- 17. Last week, Eastside Concrete Co. built a small driveway that required  $5\frac{1}{3}$  cubic yards of concrete. This week, Eastside must build another driveway that is  $2\frac{1}{2}$  times larger. How many yards of concrete will be required?  $13\frac{1}{3}$  cu yd  $2\frac{1}{2} \times 5\frac{1}{3} = \frac{5}{2} \times \frac{16}{3} = \frac{5 \times 8}{1 \times 3} = \frac{40}{3} = 13\frac{1}{3}$  cu yd
- **18.** Ted Schopf bought eight pieces of copper pipe that were each  $6\frac{3}{4}$  inches long. What was the total length of pipe that Ted bought? (Give the answer in inches.) <u>54 in.</u>

 $8 \times 6\frac{3}{4} = \frac{8}{1} \times \frac{27}{4} = \frac{2 \times 27}{1 \times 1} = 54$  in.

**19.** Linda Johanssen had  $2\frac{1}{4}$  quarts of liquid fertilizer in a container. Her supervisor asked her to mix  $\frac{2}{3}$  of the fertilizer with water and save the remainder. How many quarts of fertilizer did Linda mix with water?  $1\frac{1}{2}$  qt

 $\frac{2}{3} \text{ of } 2\frac{1}{4} = \frac{2}{3} \times 2\frac{1}{4} = \frac{2}{3} \times \frac{9}{4} = \frac{1 \times 3}{1 \times 2} = \frac{3}{2} = 1\frac{1}{2} \text{ qt}$ 

- C Business Applications and Critical Thinking. Use fractions and mixed numbers to solve each of the following. State the answers as whole numbers, proper fractions, or mixed numbers in lowest terms. (cont'd)
- 20. Landscaper Bill Holmes needs several pieces of PVC irrigation pipe, each 3 feet 4 inches long. PVC pipe comes in 20-foot lengths. How many pieces can Bill cut out of one length of pipe? (*Hint:* 4 inches equal <sup>1</sup>/<sub>3</sub> foot.) 6 pieces

   1 20 10 20 3 2×3 6

 $20 \div 3\frac{1}{3} = \frac{20}{1} \div \frac{10}{3} = \frac{20}{1} \times \frac{3}{10} = \frac{2 \times 3}{1 \times 1} = \frac{6}{1} = 6 \text{ pieces}$ 

**21.** Robert Burke has a diesel-powered generator on his ranch. The generator has a tank that holds  $3\frac{3}{4}$  gallons of diesel fuel. He stores the diesel fuel in 55-gallon drums (barrels). How many times can Robert refill his generator from one drum of fuel?  $14\frac{2}{3}$  times

$$55 \div 3\frac{3}{4} = \frac{55}{1} \div \frac{15}{4} = \frac{55}{1} \times \frac{4}{15} = \frac{11 \times 4}{1 \times 3} = \frac{44}{3} = 14\frac{2}{3}$$

**22.** Home builders Bill and John Walter are planning a narrow stairway to an attic. The stairs will each be 2 feet 8 inches long. They will cut the stairs from boards that are 8 feet long. How many whole stairs can they cut from one 8-foot board? (*Hint:* 8 inches is  $\frac{2}{3}$  foot.) <u>3 stairs</u>

$$8 \div 2\frac{2}{3} = \frac{8}{1} \div \frac{8}{3} = \frac{8}{1} \times \frac{3}{8} = \frac{1 \times 3}{1 \times 1} = 3$$
 stairs