Quadratic Equations and Inequalities

CHAPTER 9

9.4

Applications of Quadratic Equations

Objective

1 Application problems

Application problems

Example 1

A kicker punts a football at an angle of 60º with the ground. Assuming no air resistance, the height *h*, in feet, of the punted football *x* feet from where it was kicked can be given by $h = -0.0065x^2 + 1.73x + 4$. How far is the football from the kicker when the height of the football is 70 ft? Round to the nearest tenth.

Strategy:

To find the football's distance from the kicker when it is 70 ft above the ground, solve the equation $h = -0.0065x^2 + 1.73x + 4$ for *x* when *h* = 70.

Example 1 – *Solution*

$$h = -0.0065x^2 + 1.73x + 4$$

$$70 = -0.0065x^2 + 1.73x + 4$$

Replace *h* by 70.

$$0 = -0.0065x^2 + 1.73x - 66$$

Write in standard form.

$$x = \frac{-1.73 \pm \sqrt{1.73^2 - 4(-0.0065)(-66)}}{2(-0.0065)}$$

Solve by using the quadratic formula.

$$x = \frac{-1.73 \pm \sqrt{1.2769}}{-0.013} \approx \frac{-1.73 \pm 1.13}{-0.013}$$

Example 1 – *Solution*

cont'd

$$x \approx \frac{-1.73 + 1.13}{-0.013} \qquad x \approx \frac{-1.73 - 1.13}{-0.013}$$

$$x \approx 46.2 \qquad x \approx 220$$

When the football is 70 ft high, it is either 46.2 ft or 220 ft from the kicker.

Example 1 – *Solution*

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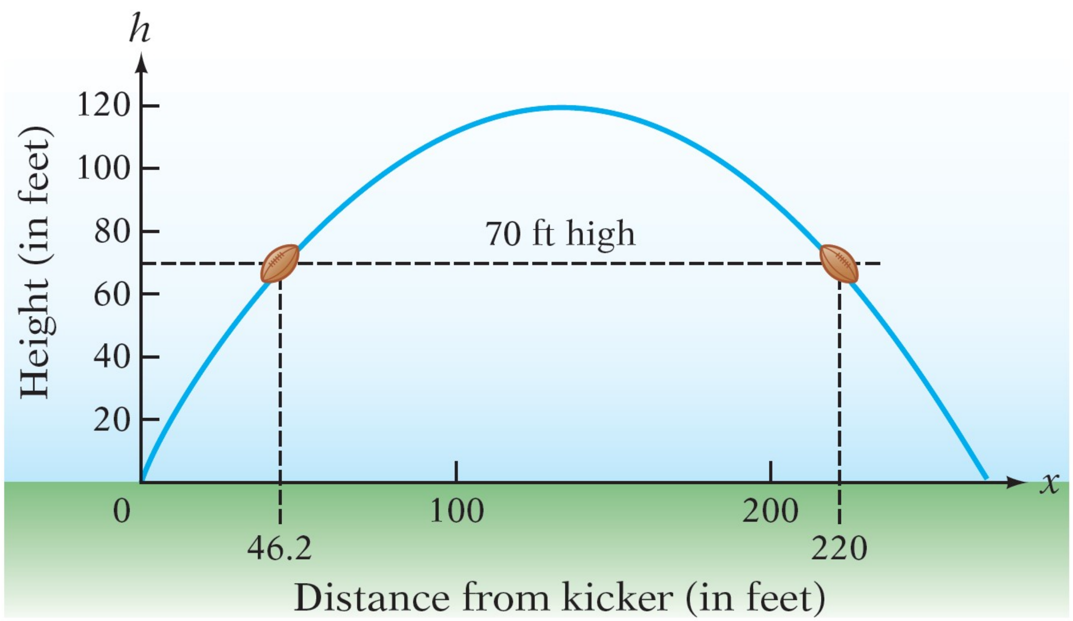
The flight of the football is shown below.



Note that it is 70 ft above the ground twice, when $x = 46.2$ ft and when $x = 220$ ft from the kicker.

Example 2

A swimming pool is being emptied using two hoses. The smaller hose takes 2 h longer to empty the pool than does the larger hose. After the valves on both hoses have been opened for 1 h, the larger hose is turned off. It takes the smaller hose 1 more hour to empty the pool. How long would it take the larger hose, working alone, to empty the pool?

Strategy:

- This is a work problem.
- The unknown time for the larger hose working alone: t
- The unknown time for the smaller hose working alone: $t + 2$

Example 2

cont'd

- The larger hose operates for 1 h; the smaller hose operates for 2 h.

	Rate	$\cdot$	Time	$=$	Part
Larger hose	$\frac{1}{t}$	$\cdot$	1	$=$	$\frac{1}{t}$
Smaller hose	$\frac{1}{t+2}$	$\cdot$	2	$=$	$\frac{2}{t+2}$

- The sum of the part of the task completed by the larger hose and the part completed by the smaller hose is 1.

Example 2 – *Solution*

$$\frac{1}{t} + \frac{2}{t+2} = 1$$

$$t(t+2)\left(\frac{1}{t} + \frac{2}{t+2}\right) = t(t+2) \cdot 1$$

Multiply each side of the equation by the LCD.

$$(t+2) + 2t = t^2 + 2t$$

Simplify.

$$0 = t^2 - t - 2$$

Write the quadratic equation in standard form.

$$0 = (t+1)(t-2)$$

Factor.

$$t + 1 = 0 \qquad t - 2 = 0$$

Use the Principle of Zero Products.

$$t = -1 \qquad t = 2$$

Example 2 – *Solution*

cont'd

Because time cannot be negative, $t = -1$ is not possible.

It would take the larger hose, working alone, 2 h to empty the pool.