CHAPTER

8

Rational Exponents and Radicals

8.3 Radical Functions

Objectives

1. Find the domain of a radical function

2. Graph a radical function

Find the domain of a radical function

Find the domain of a radical function

A **radical function** is one that contains a fractional exponent or a variable underneath a radical. Examples of radical functions are shown below.

$$f(x) = 3\sqrt[4]{x^5} - 7$$
$$g(x) = 3x - 2x^{\frac{1}{2}} + 5$$

Note that these are *not* polynomial functions because polynomial functions do not contain variables raised to a fractional power or variable radical expressions.

Find the domain of a radical function

The domain of a radical function is a set of real numbers for which the radical expression is a real number.

For example, –9 is one number that would be excluded from the domain of $f(x) = \sqrt{x + 5}$ because

$f(-9) = \sqrt{-9 + 5} = \sqrt{-4}$, which is not a real number.

Find the domain of a radical function

If the index of a radical expression is an even number, the radicand must be greater than or equal to zero to ensure that the value of the radical expression will be a real number.

If the index of a radical expression is an odd number, the radicand may be a positive or a negative number.

Example 1

State the domain of each function in set-builder notation.

A. $V(x) = \sqrt[4]{6 - 4x}$ **B.** $R(x) = \sqrt[5]{x + 4}$

Solution:

A. $6 - 4x \geq 0$

$-4x \geq -6$

$x \leq \dfrac{3}{2}$

V contains an even root. Therefore, the radicand must be greater than or equal to zero.

The domain is $\{x | x \leq \frac{3}{2}\}$.

B. $R(x) = \sqrt[5]{x + 4}$

Because *R* contains an odd root, the radicand may be positive or negative.

The domain is $\{x | x \in \text{real numbers}\}$.

Graph a radical function

Graph a radical function

The graph of a radical function is produced in the same manner as the graph of any other function.

The function is evaluated at several values in the domain of the function, and the resulting ordered pairs are graphed.

Ordered pairs must be graphed until an accurate graph can be drawn.

Example 2

Graph: $H(x) = \sqrt[3]{x}$

Solution:

Because *H* contains only an odd root, the domain of *H* is all real numbers. Choose some values of *x* in the domain of *H*, and evaluate the function for those values.

Some possible choices are given in the following table.

x	$y = \sqrt[3]{x}$
-8	-2
-1	-1
0	0
1	1
8	2