Conic Sections

CHAPTER 12

12.4 Solving Nonlinear Systems of Equations

Objective

1 Solve nonlinear systems of equations

Solving Nonlinear Systems of Equations

Solving Nonlinear Systems of Equations

A **nonlinear system of equations** is one in which one or more equations of the system is not a linear equation.

Nonlinear systems of equations can be solved by using either a substitution method or an addition method.

Example 1

Solve: $y = 2x^2 - 3x - 1$
$y = x^2 - 2x + 5$

Solution:

(1) $y = 2x^2 - 3x - 1$

(2) $y = x^2 - 2x + 5$

$$2x^2 - 3x - 1 = x^2 - 2x + 5$$

Use the substitution method to solve for *x*.

$$x^2 - x - 6 = 0$$

$$(x + 2)(x - 3) = 0$$

$$x + 2 = 0 \qquad x - 3 = 0$$

$$x = -2 \qquad x = 3$$

Example 1 – *Solution*

cont'd

$$y = 2x^2 - 3x - 1$$

$$y = 2(-2)^2 - 3(-2) - 1$$

Substitute each value of x into equation (1) or equation (2) and solve for y. We will use equation (1).

$$y = 8 + 6 - 1$$

$$y = 13$$

When $x = -2$, $y = 13$.

$$y = 2x^2 - 3x - 1$$

$$y = 2(3)^2 - 3(3) - 1$$

$$y = 18 - 9 - 1$$

$$y = 8$$

When $x = 3$, $y = 8$.

The solutions are (–2, 13) and (3, 8).

Example 2

Solve: $3x^2 - 2y^2 = 26$
$x^2 - y^2 = 5$

Solution:

(1) $3x^2 - 2y^2 = 26$

(2) $x^2 - y^2 = 5$

$$\begin{array}{r} 3x^2 - 2y^2 = 26 \\ -2x^2 + 2y^2 = -10 \\ \hline x^2 = 16 \end{array}$$

$$x = \pm 4$$

Use the addition method. We will eliminate *y*. Multiply equation (2) by −2 and solve for *x*.

Example 2 – *Solution*

cont'd

$$x^2 - y^2 = 5$$

$$(-4)^2 - y^2 = 5$$

$$16 - y^2 = 5$$

$$-y^2 = -11$$

$$y^2 = 11$$

$$y = \pm\sqrt{11}$$

Substitute each value of x into equation (1) or equation (2) and solve for y. We will use equation (2).

When $x = -4$, $y = -\sqrt{11}$ or $y = \sqrt{11}$.

Example 2 – *Solution*

cont'd

$$x^2 - y^2 = 5$$

$$4^2 - y^2 = 5$$

$$16 - y^2 = 5$$

$$-y^2 = -11$$

$$y^2 = 11$$

$$y = \pm\sqrt{11}$$

When $x = 4$, $y = -\sqrt{11}$ or $y = \sqrt{11}$.

The solutions are $(-4, -\sqrt{11})$, $(-4, \sqrt{11})$, $(4, -\sqrt{11})$, and $(4, \sqrt{11})$.