Conic Sections

CHAPTER

12

12.3 The Ellipse and the Hyperbola

Objectives

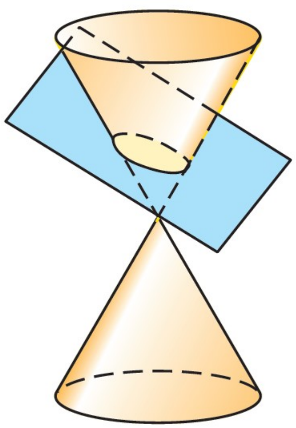
1 Graph an ellipse with center at the origin

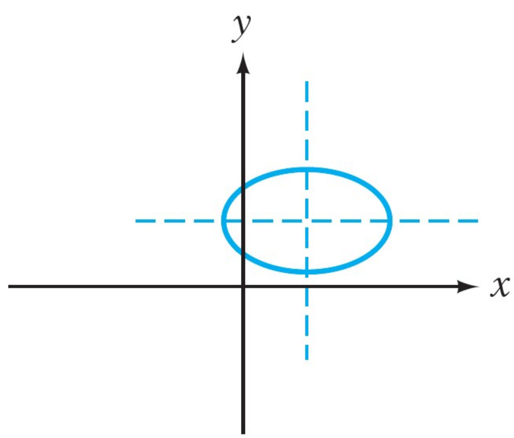
2 Graph a hyperbola with center at the origin

Graph an ellipse with center at the origin

Graph an ellipse with center at the origin

The orbits of the planets around the sun are “oval” shaped. This oval shape can be described as an **ellipse,** which is another of the conic sections.

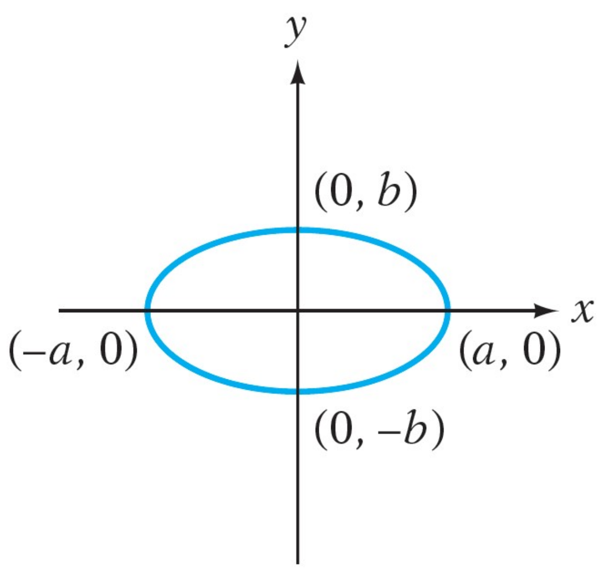




An ellipse has two **axes of symmetry.** The intersection of these two axes is the **center** of the ellipse.

Graph an ellipse with center at the origin

An ellipse with center at the origin is shown at the right. Note that there are two *x*-intercepts and two *y*-intercepts.



Graph an ellipse with center at the origin

STANDARD FORM OF THE EQUATION OF AN ELLIPSE WITH CENTER AT THE ORIGIN

The equation of an ellipse with center at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The coordinates of the x-intercepts are $(a, 0)$ and $(-a, 0)$. The coordinates of the y-intercepts are $(0, b)$ and $(0, -b)$.

By finding the coordinates of the *x*- and *y*-intercepts of an ellipse and using the fact that the ellipse is “oval” shaped, we can sketch a graph of an ellipse.

Example 1

Sketch a graph of the ellipse given by the equation.

A. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ **B.** $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

Solution:

A. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$a^2 = 9, b^2 = 4$

x-intercepts:
$(3, 0)$ and $(-3, 0)$

The coordinates of the x-intercepts are $(a, 0)$ and $(-a, 0)$.

y-intercepts:
$(0, 2)$ and $(0, -2)$

The coordinates of the y-intercepts are $(0, b)$ and $(0, -b)$.

Example 1 – *Solution*

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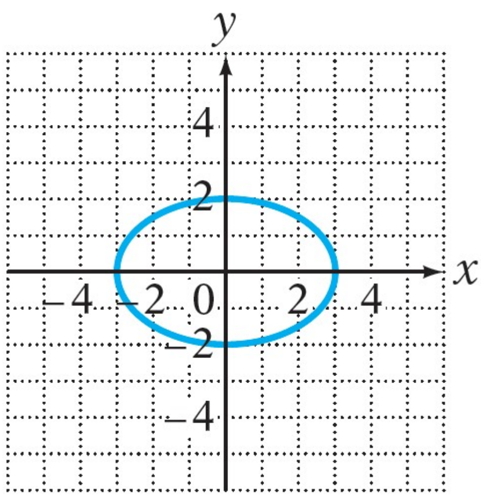


Use the intercepts and symmetry to sketch the graph of the ellipse.

B. $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

$a^2 = 16, b^2 = 12$

x-intercepts:
$(4, 0)$ and $(-4, 0)$

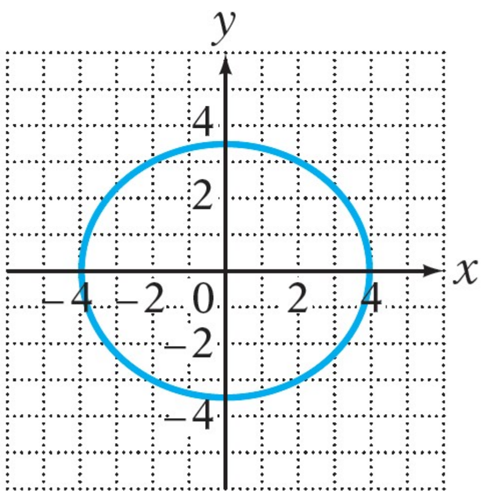
The coordinates of the x-intercepts are $(a, 0)$ and $(-a, 0)$.

Example 1 – *Solution*

cont'd

y-intercepts:
$(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$

The coordinates of the y-intercepts are $(0, b)$ and $(0, -b)$.

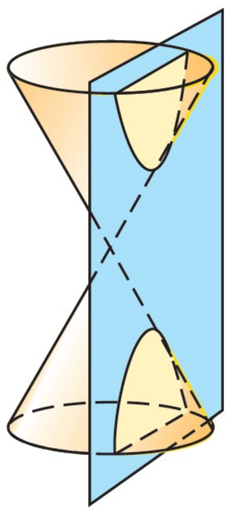


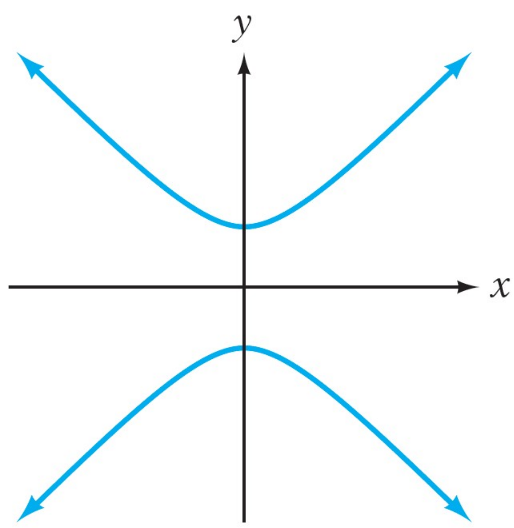
Use the intercepts and symmetry to sketch the graph of the ellipse.
$2\sqrt{3} \approx 3.5$

Graph a hyperbola with center at the origin

Graph a hyperbola with center at the origin

A **hyperbola** is a conic section that is formed by the intersection of a right circular cone and a plane perpendicular to the base of the cone.

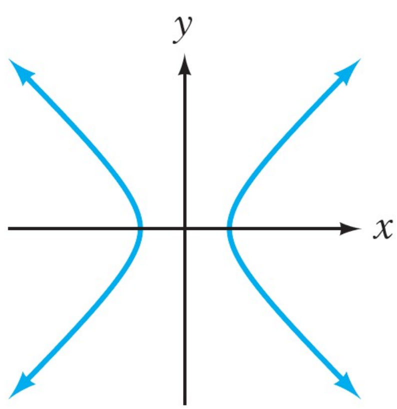


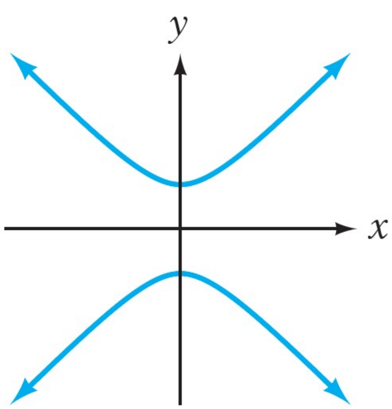


Graph a hyperbola with center at the origin

The hyperbola has two **vertices** and an **axis of symmetry** that passes through the vertices. The **center** of a hyperbola is the point halfway between the vertices.

The graphs below show two graphs of a hyperbola with center at the origin.





Graph a hyperbola with center at the origin

In the first graph, the axis of symmetry that contains the vertices is the *x*-axis.

In the second graph, the axis of symmetry that contains the vertices is the *y*-axis.

Note that in either case, the graph of a hyperbola is not the graph of a function. The graph of a hyperbola is the graph of a relation.

Graph a hyperbola with center at the origin

STANDARD FORM OF THE EQUATION OF A HYPERBOLA WITH CENTER AT THE ORIGIN

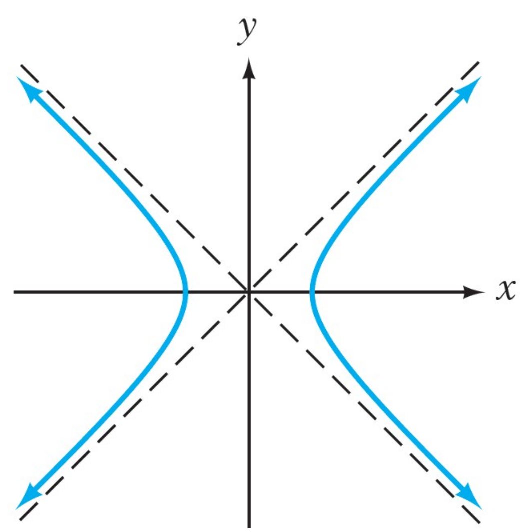
The equation of a hyperbola for which the axis of symmetry that contains the vertices is the x-axis is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. The coordinates of the vertices are $(a, 0)$ and $(-a, 0)$.

The equation of a hyperbola for which the axis of symmetry that contains the vertices is the y-axis is $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$. The coordinates of the vertices are $(0, b)$ and $(0, -b)$.

Graph a hyperbola with center at the origin

To sketch a hyperbola, it is helpful to draw two lines that are "approached" by the hyperbola. These two lines are called **asymptotes.**

As a point on the hyperbola gets farther from the origin, the hyperbola "gets closer to" the asymptotes.



Because the asymptotes are straight lines, their equations are linear equations.

The equations of the asymptotes of a hyperbola with center at the origin are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

Example 2

Sketch a graph of the hyperbola given by the equation.

A. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$ **B.** $\dfrac{y^2}{16} - \dfrac{x^2}{25} = 1$

Solution:

A. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

$a^2 = 16, b^2 = 4$

Axis of symmetry:
x-axis

Vertices:
$(4, 0)$ and $(-4, 0)$

The coordinates of the vertices are $(a, 0)$ and $(-a, 0)$.

Asymptotes:

$y = \dfrac{1}{2}x$ and $y = -\dfrac{1}{2}x$

The equations of the asymptotes are

$y = \dfrac{b}{a}x$ and $y = -\dfrac{b}{a}x$.

Example 2 – *Solution*

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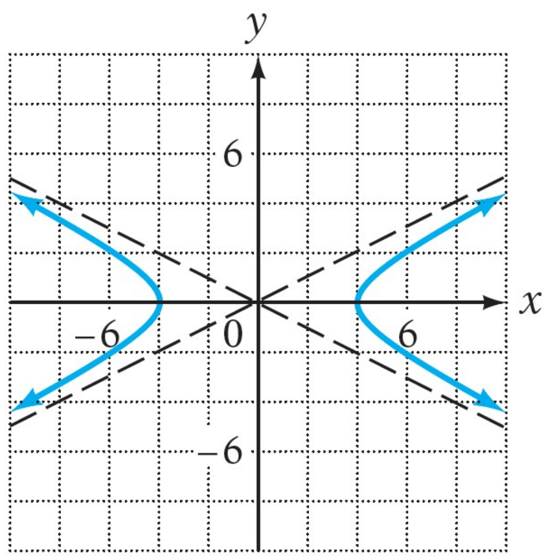


Sketch the asymptotes. Use symmetry and the fact that the hyperbola will approach the asymptotes to sketch its graph.

B. $\dfrac{y^2}{16} - \dfrac{x^2}{25} = 1$

$a^2 = 25, b^2 = 16$

Axis of symmetry:
y-axis

Example 2 – *Solution*

cont'd

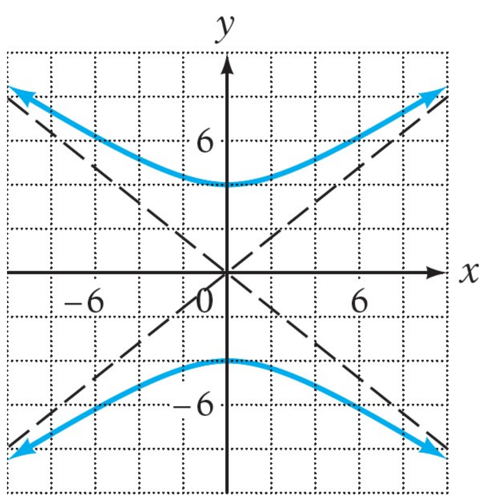
Vertices:
(0, 4) and (0, −4)

The coordinates of the vertices are (0, b) and (0, $-b$).

Asymptotes:

$$y = \frac{4}{5}x \text{ and } y = -\frac{4}{5}x$$

The equations of the asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.



Sketch the asymptotes. Use symmetry and the fact that the hyperbola will approach the asymptotes to sketch its graph.