Conic Sections

CHAPTER 12

12.1 The Parabola

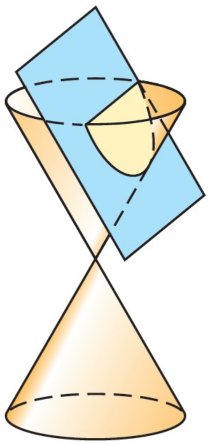
Objective

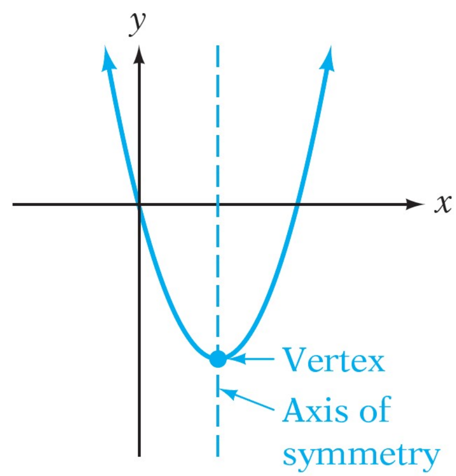
1 Graph parabolas

Graph parabolas

Graph parabolas

The **conic sections** are curves that can be constructed from the intersection of a plane and a right circular cone. The four conic sections are the parabola, circle, ellipse, and hyperbola.





A **parabola** is a conic section formed by the intersection of a right circular cone and a plane parallel to the side of the cone.

Graph parabolas

Every parabola has an **axis of symmetry** and a **vertex** that is on the axis of symmetry.

To understand the axis of symmetry, think of folding the paper along that axis. The two halves of the curve will match up.

The graph of the equation $y = ax^2 + bx + c,\ a \neq 0,$ a parabola with the axis of symmetry parallel to the *y*-axis.

Graph parabolas

The parabola opens up when $a > 0$ and opens down when $a < 0$.

When the parabola opens up, the vertex is the lowest point on the parabola. When the parabola opens down, the vertex is the highest point on the parabola.

The coordinates of the vertex can be found by completing the square.

Graph parabolas

By following the procedure of completing the square on the equation $y = ax^2 + bx + c$, we can find that the ***x*-coordinate of the vertex** is $-\frac{b}{2a}$.

The *y*-coordinate of the vertex can be determined by substituting this value of *x* into $y = ax^2 + bx + c$ and solving for *y*.

Because the axis of symmetry is parallel to the *y*-axis and passes through the vertex, the equation of the **axis of symmetry** is $x = -\frac{b}{2a}$.

Example 1

Find the coordinates of the vertex and the equation of the axis of symmetry of the parabola given by the equation $y = x^2 + 2x - 3$. Then sketch the graph of the parabola.

Solution:

$$-\frac{b}{2a} = -\frac{2}{2(1)}$$

The x-coordinate of the vertex is $-\frac{b}{2a}$.
$a = 1$, $b = 2$

$$= -1$$

$$y = x^2 + 2x - 3$$

Example 1 – *Solution*

cont'd

$$y = (-1)^2 + 2(-1) - 3$$

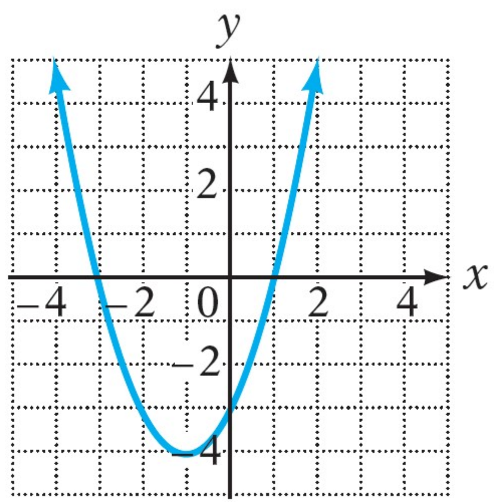
$$y = -4$$

Find the y-coordinate of the vertex by replacing x by -1 and solving for y.

The coordinates of the vertex are $(-1, -4)$.

The equation of the axis of symmetry is $x = -1$.

The equation of the axis of symmetry is $x = -\frac{b}{2a}$.

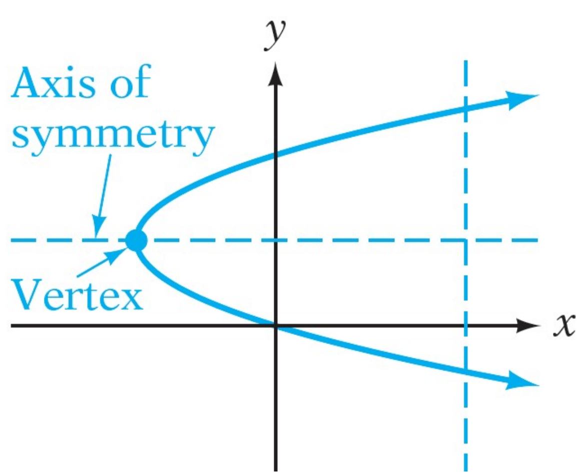


Because a is positive, the parabola opens up. Use the vertex and axis of symmetry to sketch the graph.

Graph parabolas

The graph of an equation of the form $x = ay^2 + by + c$, $a \neq 0$, is also a parabola.

In this case, the parabola opens to the right when *a* is positive and opens to the left when *a* is negative.



Graph parabolas

For a parabola of this form, the ***y*-coordinate of the vertex** is $-\frac{b}{2a}$. The **axis of symmetry** is the line whose equation is $y = -\frac{b}{2a}$.

The vertical line test reveals that the graph of a parabola of this form is not the graph of a function. The graph of $x = ay^2 + by + c$ is the graph of a relation.