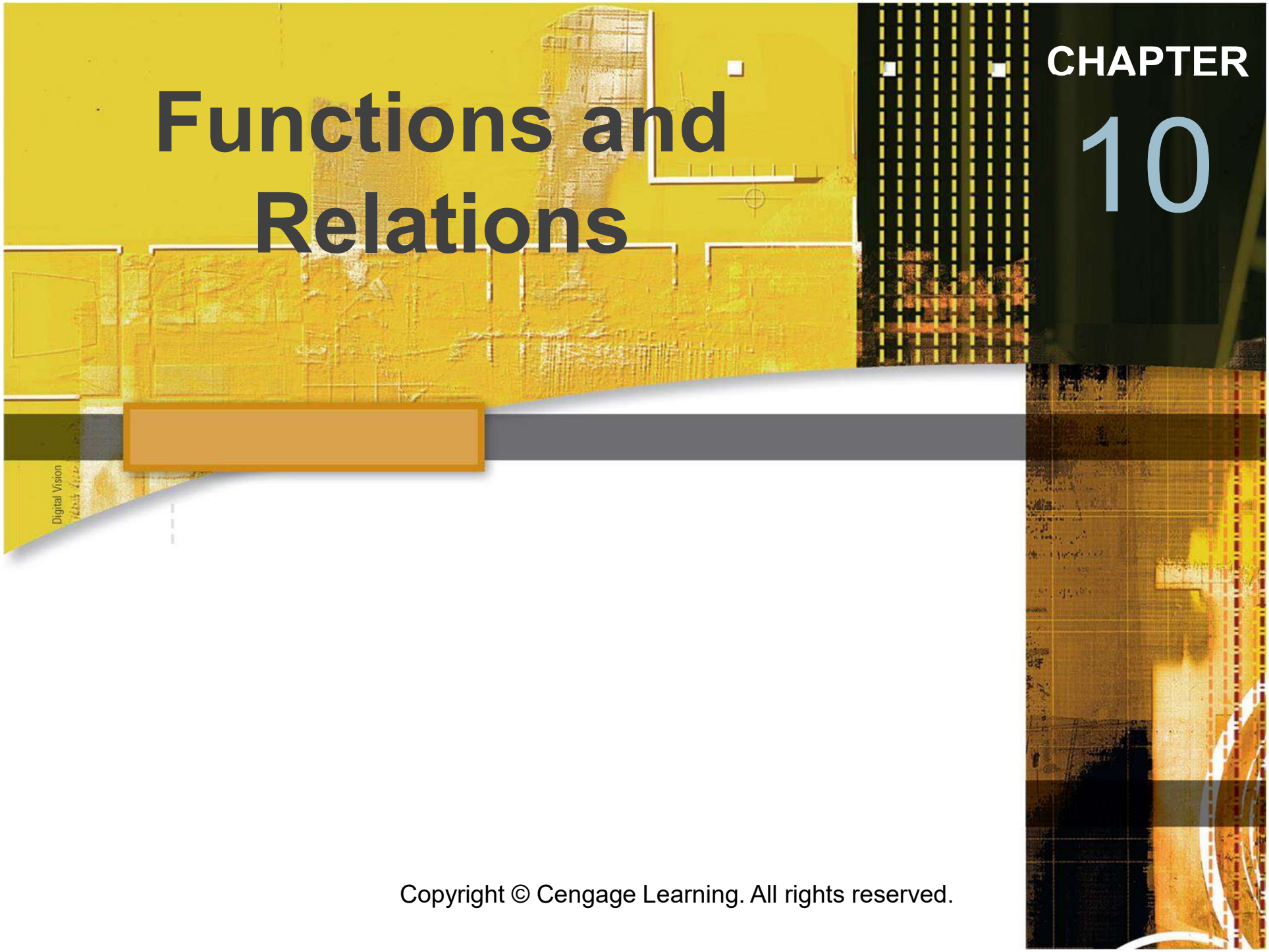

10.2 Algebra of Functions

Objectives

1. Perform operations on functions

2. Find the composition of two functions

Perform operations on functions

Perform operations on functions

The operations of addition, subtraction, multiplication, and division of functions are defined as follows:

OPERATIONS ON FUNCTIONS

If f and g are functions and x is an element of the domain of each function, then

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

Example 1

Given $f(x) = x^2 - x + 1$ and $g(x) = x^3 - 4$, find $(f - g)(3)$.

Solution:

$$(f - g)(3) = f(3) - g(3)$$

$$= (3^2 - 3 + 1) - (3^3 - 4)$$

$$= 7 - 23$$

$$(f - g)(3) = -16$$

Example 3

Given $f(x) = x^2 + 4x + 4$ and $g(x) = x^3 - 2$, find $\left(\frac{f}{g}\right)(3)$.

Solution:

$$\left(\frac{f}{g}\right)(3) = \frac{f(3)}{g(3)}$$

$$= \frac{3^2 + 4(3) + 4}{3^3 - 2}$$

$$= \frac{25}{25}$$

$$\left(\frac{f}{g}\right)(3) = 1$$

Find the composition of two functions

Find the composition of two functions

Composition of functions is another way in which functions can be combined. This method of combining functions uses the output of one function as the input for a second function.

DEFINITION OF THE COMPOSITION OF TWO FUNCTIONS

Let f and g be two functions such that $g(x)$ is in the domain of f for all x in the domain of g. Then the **composition** of the two functions, denoted by $f \circ g$, is the function whose value at x is given by $(f \circ g)(x) = f[g(x)]$.

Find the composition of two functions

The function defined by $(f \circ g)(x)$ is called the **composite** of f and g. We read $(f \circ g)(x)$ as "f circle g of x," and we read $f[g(x)]$ as "f of g of x."

In general, composition of functions is *not* a commutative operation: $f[g(x)] \neq g[f(x)]$.

Example 4

Given $f(x) = x^2 - 1$ and $g(x) = 3x + 4$, find each composite function.

A. $f[g(2)]$ **B.** $g[f(x)]$

Solution:

A. $g(x) = 3x + 4$

$g(2) = 3(2) + 4 = 10$ Find $g(2)$.

$f(x) = x^2 - 1$

$f[g(2)] = [g(2)]^2 - 1$ Find $f[g(2)]$.

Example 4 – *Solution*

cont'd

$$= 10^2 - 1$$

$$= 100 - 1$$

$$f[g(2)] = 99$$

B. $g(x) = 3x + 4$

$$g[f(x)] = 3[f(x)] + 4$$

Replace *x* by *f*(*x*).

Example 4 – *Solution*

cont'd

$= 3[x^2 - 1] + 4$ $f(x) = x^2 - 1$

$= 3x^2 - 3 + 4$ **Simplify.**

$g[f(x)] = 3x^2 + 1$