

Factoring

CHAPTER

6

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6.3

Factoring Polynomials of the Form $ax^2 + bx + c$

Objectives

- 1 Factor trinomials of the form $ax^2 + bx + c$ by using trial factors
- 2 Factor trinomials of the form $ax^2 + bx + c$ by grouping



Factor trinomials of the form
 $ax^2 + bx + c$ by using trial factors



Factor trinomials of the form $ax^2 + bx + c$ by using trial factors

To **factor a trinomial of the form $ax^2 + bx + c$** means to express the polynomial as the product of two binomials.

Factoring such polynomials by trial and error may require testing many trial factors.

Factor trinomials of the form $ax^2 + bx + c$ by using trial factors

To reduce the number of trial factors, remember the following points.

POINTS TO REMEMBER IN FACTORING $ax^2 + bx + c$

1. If the terms of the trinomial have a common factor, factor out the common factor first.
2. If the terms of the trinomial do not have a common factor, then the terms of a binomial factor cannot have a common factor.
3. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign as the coefficient of x in the trinomial.
4. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.

Example 1

Factor: $3x^2 + 20x + 12$

Solution:

Factors of 3	Factors of 12
1, 3	1, 12 2, 6 3, 4

Trial Factors	Middle Term
$(x + 3)(3x + 4)$	$4x + 9x = 13x$
$(3x + 1)(x + 12)$	$36x + x = 37x$
$(3x + 2)(x + 6)$	$18x + 2x = \mathbf{20x}$

Because 20 is positive, only the positive factors of 12 need be tried.

Write the trial factors.
Use FOIL to check the middle term.

Example 1 – *Solution*

cont'd

$$3x^2 + 20x + 12 = (3x + 2)(x + 6)$$

Check:

$$\begin{aligned}(3x + 2)(x + 6) &= 3x^2 + 18x + 2x + 12 \\ &= 3x^2 + 20x + 12\end{aligned}$$



Factor trinomials of the form $ax^2 + bx + c$ by using trial factors

The first step in factoring a trinomial is to determine whether its terms have a common factor. If so, factor out the GCF of the terms.

Example 4

Factor: $3x^3 - 23x^2 + 14x$

Solution:

The GCF of $3x^3$, $23x^2$, and $14x$ is x .

Find the GCF of the terms of the polynomial.

$$3x^3 - 23x^2 + 14x = x(3x^2 - 23x + 14)$$

Factor out the GCF.

Factors of 3	Factors of 14
1, 3	-1, -14 -2, -7

Factor the trinomial $3x^2 - 23x + 14$.

Example 4 – *Solution*


cont'd

Trial Factors	Middle Term
$(x - 1)(3x - 14)$	$-14x - 3x = -17x$
$(x - 14)(3x - 1)$	$-x - 42x = -43x$
$(x - 2)(3x - 7)$	$-7x - 6x = -13x$
$(x - 7)(3x - 2)$	$-2x - 21x = -\mathbf{23x}$

$$3x^3 - 23x^2 + 14x = x(x - 7)(3x - 2)$$

Check:

$$\begin{aligned}x(x - 7)(3x - 2) &= x(3x^2 - 2x - 21x + 14) \\ &= x(3x^2 - 23x + 14) \\ &= 3x^3 - 23x^2 + 14x\end{aligned}$$



Factor trinomials of the form
 $ax^2 + bx + c$ by grouping

Factor trinomials of the form $ax^2 + bx + c$ by grouping

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b .

Use the two factors to rewrite the middle term of the trinomial as the sum of two terms.

Then use factoring by grouping to write the factorization of the trinomial.

Example 5

Factor: $2x^2 + 19x - 10$

Solution:

$$a \cdot c = 2(-10) = -20$$

Find $a \cdot c$

$$-1(20) = -20$$

Find two numbers whose product is -20 and whose sum is 19 .

$$-1 + 20 = 19$$

$$2x^2 + 19x - 10 = 2x^2 - x + 20x - 10$$

Rewrite $19x$ as $-x + 20x$.

Example 5 – *Solution*

cont'd

$$= (2x^2 - x) + (20x - 10) \quad \text{Factor by grouping.}$$

$$= x(2x - 1) + 10(2x - 1)$$

$$= (2x - 1)(x + 10)$$



Factor trinomials of the form $ax^2 + bx + c$ by grouping

Remember that the first step in factoring a trinomial is to determine whether the terms have a common factor. If so, factor out the GCF of the terms.

Example 7

Factor: $24x^2y - 76xy + 40y$

Solution:

$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$ The terms of the polynomial have a common factor, $4y$. Factor out the GCF.

$$a \cdot c = 6(10) = 60$$

$$-4(-15) = 60$$

$$-4 + (-15) = -19$$

To factor $6x^2 - 19x + 10$, first find $a \cdot c$.

Find two numbers whose product is 60 and whose sum is -19 .

Example 7 – *Solution*

cont'd

$$6x^2 - 19x + 10 = 6x^2 - 4x - 15x + 10$$

Rewrite $-19x$ as $-4x - 15x$.

$$= (6x^2 - 4x) - (15x - 10)$$

Factor by grouping.

$$= 2x(3x - 2) - 5(3x - 2)$$

$$= (3x - 2)(2x - 5)$$

$$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$$

Write the complete factorization of the given polynomial.

$$= 4y(3x - 2)(2x - 5)$$