Factoring

CHAPTER 6

6.2 Factoring Polynomials of the Form $x^2 + bx + c$

Objectives

1 Factor trinomials of the form $x^2 + bx + c$

2 Factor completely

Factor trinomials of the form

$x^2 + bx + c$

Factor trinomials of the form $x^2 + bx + c$

Trinomials of the form $x^2 + bx + c$, where b and c are integers, are shown below.

$x^2 + 9x + 14$, $b = 9$, $c = 14$

$x^2 - x - 12$, $b = -1$, $c = -12$

$x^2 - 2x - 15$, $b = -2$, $c = -15$

Some trinomials expressed as the product of binomials are shown at the right. They are in factored form.

Trinomial		Factored Form
$x^2 + 9x + 14$	$=$	$(x + 2)(x + 7)$
$x^2 - x - 12$	$=$	$(x + 3)(x - 4)$
$x^2 - 2x - 15$	$=$	$(x + 3)(x - 5)$

Factor trinomials of the form $x^2 + bx + c$

POINTS TO REMEMBER IN FACTORING $x^2 + bx + c$

1. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.
2. In the trinomial, the constant term is the product of the constant terms of the binomials.
3. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign as the coefficient of x in the trinomial.
4. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.

Example 1

Factor: $x^2 + 18x + 32$

Solution:

Factors of 32	Sum
1, 32	33
2, 16	**18**
4, 8	12

Try only positive factors of 32 [Point 3].

Once the correct pair is found,
the other factors need not be tried.

$x^2 + 18x + 32 = (x + 2)(x + 16)$ Write the factors of the trinomial.

Check:

$(x + 2)\,(x + 16) = x^2 + 16x + 2x + 32$

$= x^2 + 18x + 32$

Factor trinomials of the form $x^2 + bx + c$

Not all trinomials can be factored when using only integers. Consider the trinomial $x^2 - 6x - 8$.

Factors of -8	Sum
$1, -8$	-7
$-1, 8$	7
$2, -4$	-2
$-2, 4$	2

Because none of the pairs of factors of -8 has a sum of -6, the trinomial is not factorable using integers.

The trinomial is said to be **nonfactorable over the integers.**

Factor completely

Factor completely

A polynomial is **factored completely** when it is written as a product of factors that are nonfactorable over the integers.

The first step in *any* factoring problem is to determine whether the terms of the polynomial have a *common factor*. If they do, factor it out first.

Example 3

Factor: $3x^3 + 15x^2 + 18x$

Solution:

The GCF of $3x^3$, $15x^2$, and $18x$ is $3x$.

Find the GCF of the terms of the polynomial.

$3x^3 + 15x^2 + 18x = 3x(x^2) + 3x(5x) + 3x(6)$

Factor out the GCF.

$= 3x(x^2 + 5x + 6)$

Write the polynomial as a product of factors.

Factors of 6	Sum
1, 6	7
2, 3	**5**

Factor the trinomial $x^2 + 5x + 6$. Try only positive factors of 6.

Example 3 – *Solution*

cont'd

$$3x^3 + 15x^2 + 18x = 3x(x + 2)(x + 3)$$

Check:

$$3x(x + 2)\,(x + 3) = 3x(x^2 + 3x + 2x + 6)$$

$$= 3x(x^2 + 5x + 6)$$

$$= 3x^3 + 15x^2 + 18x$$