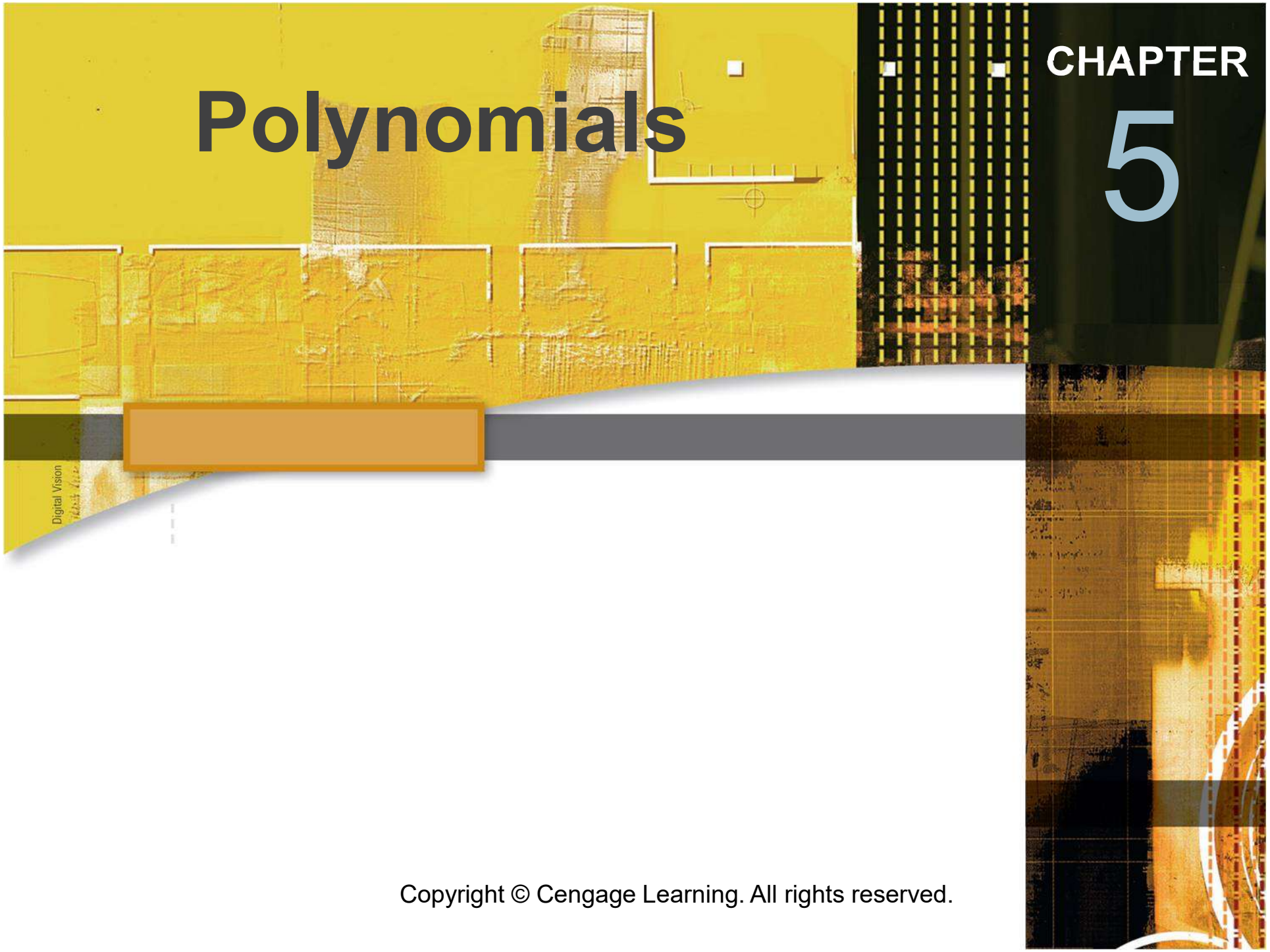
Polynomials

CHAPTER 5



5.3 Multiplication of Polynomials

Objectives

1. Multiply a polynomial by a monomial
2. Multiply two polynomials
3. Multiply two binomials
4. Multiply binomials that have special products
5. Application problems

Multiply a polynomial by a monomial

Multiply a polynomial by a monomial

To multiply a polynomial by a monomial, use the distributive Property and the Rule for multiplying Exponential Expressions.

Example 1

Multiply.

A. $-2x(x^2 - 4x - 3)$ **B.** $(5x + 4)(-2x)$ **C.** $x^3(2x^2 - 3x + 2)$

Solution:

A. $-2x(x^2 - 4x - 3)$

$= -2x(x^2) - (-2x)(4x) - (-2x)(3)$ Use the Distributive Property.

$= -2x^3 + 8x^2 + 6x$ Use the Rule for Multiplying Exponential Expressions.

Example 1 – *Solution*

cont'd

B. $(5x + 4)(-2x)$

$= 5x(-2x) + 4(-2x)$ Use the Distributive Property.

$= -10x^2 - 8x$ Use the Rule for Multiplying Exponential Expressions.

C. $x^3(2x^2 - 3x + 2)$

$= 2x^5 - 3x^4 + 2x^3$ Use the Distributive Property and the Rule for Multiplying Exponential Expressions.

Multiply two polynomials

Multiply two polynomials

Multiplication of two polynomials requires the repeated application of the Distributive Property.

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

Example 2

Multiply:
$(2b^3 - b + 1)(2b + 3)$

Solution:

$$2b^3 - b + 1$$
$$2b + 3$$

$$6b^3 - 3b + 3$$
$$4b^4 - 2b^2 + 2b$$

$$4b^4 + 6b^3 - 2b^2 - b + 3$$

Multiply $2b^3 - b + 1$ by 3.

Multiply $2b^3 - b + 1$ by $2b$. Arrange the terms in descending order.

Add the terms in each column.

Multiply two binomials

Multiply two binomials

It is often necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property.

The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast.

Example 4

Multiply:
$(4x - 3)(3x - 2)$

Solution:

$(4x - 3)(3x - 2)$

F O I L

$= 4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2)$ **Use the FOIL method.**

$= 12x^2 - 8x - 9x + 6$

$= 12x^2 - 17x + 6$ Combine like terms.

Multiply binomials that have special products

Multiply binomials that have special products

The expression $(a + b)(a – b)$ is the product of the **sum and difference of two terms.** The first binomial in the expression is a sum; the second is a difference. The two terms are *a* and *b*. The first term in each binomial is *a*. The second term in each binomial is *b*.

The expression $(a + b)^2$ is the **square of a binomial**. The first term in the binomial is *a*. The second term in the binomial is *b*.

Multiply binomials that have special products

Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

THE SUM AND DIFFERENCE OF TWO TERMS

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

Square of first term

Square of second term

Example 6

Multiply:
$(2x + 3)(2x - 3)$

Solution:

$(2x + 3)(2x - 3)$ — $(2x + 3)(2x - 3)$ is the product of the sum and difference of two terms.

$= (2x)^2 - 3^2$ — Square the first term. Square the second term.

$= 4x^2 - 9$ — Simplify.

Multiply binomials that have special products

THE SQUARE OF A BINOMIAL

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Square of first term

Twice the product of the two terms

Square of last term

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

Square of first term

Twice the product of the two terms

Square of last term

Example 7

Multiply:
$(4c + 5d)^2$

Solution:

$(4c + 5d)^2$ — $(4c + 5d)^2$ is the square of a binomial.

$= (4c)^2 + 2(4c)(5d) + (5d)^2$ — Square the first term. Find twice the product of the two terms. Square the second term.

$= 16c^2 + 40cd + 25d^2$ — Simplify.

Multiply binomials that have special products

Note that the result in Example 7 is the same result we would get by multiplying the binomial times itself and using the FOIL method.

$(4c + 5d)^2 = (4c + 5d)(4c + 5d)$

$= 16c^2 + 20cd + 20cd + 25d^2$

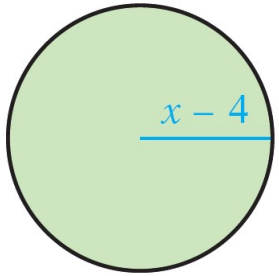
$= 16c^2 + 40cd + 25d^2$

Either method can be used to square a binomial.

Application problems

Example 9

The radius of a circle is $(x - 4)$ft. Find the area of the circle in terms of the variable x. Leave the answer in terms of π.



Strategy :

To find the area, replace the variable r in the formula $A = \pi r^2$ with the given value. Simplify the expression on the right side of the equation.

Example 9 – *Solution*

$A = \pi r^2$

$A = \pi (x - 4)^2$ This is the square of a binomial.

$A = \pi (x^2 - 8x + 16)$ Square the binomial $x - 4$.

$A = \pi x^2 - 8\pi x + 16\pi$ Use the Distributive Property.

The area is $(\pi x^2 - 8\pi x + 16\pi)$ ft^2.