Systems of Equations and Inequalities

CHAPTER 4

4.4 Application Problems

Objectives

1. Rate-of-wind and rate-of-current problems
2. Application problems

Rate-of-wind and rate-of-current problems

Rate-of-wind and rate-of-current problems

Solving motion problems that involve an object moving with or against a wind or current normally requires two variables.

STRATEGY FOR SOLVING RATE-OF-WIND AND RATE-OF-CURRENT PROBLEMS

- ► Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $rt = d$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

- ► Determine how the expressions for the distance are related, and write a system of equations.

Example 1

Flying with the wind, a plane flew 1000 mi in 5 h. Flying against the wind, the plane could fly only 500 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

Strategy:

- Rate of the plane in still air: *p*

  Rate of the wind: *w*

	Rate	Time	Distance
With wind	$p + w$	5	$5(p + w)$
Against wind	$p - w$	5	$5(p - w)$

Example 1

cont'd

- The distance traveled with the wind is 1000 mi.
  The distance traveled against the wind is 500 mi.

Solution:

$$5(p + w) = 1000$$ Write a system of equations.

$$5(p - w) = 500$$

$$\begin{aligned} p + w &= 200 \\ p - w &= 100 \end{aligned}$$ Multiply each side of both equations by $\frac{1}{5}$.

$$2p = 300$$ Add the equations.

$$p = 150$$ Solve for p.

Example 1 – *Solution*

cont'd

$$p + w = 200$$

Substitute the value of *p* into one of the equations and solve for *w*.

$$150 + w = 200$$

$$w = 50$$

The rate of the plane in calm air is 150 mph.

The rate of the wind is 50 mph.

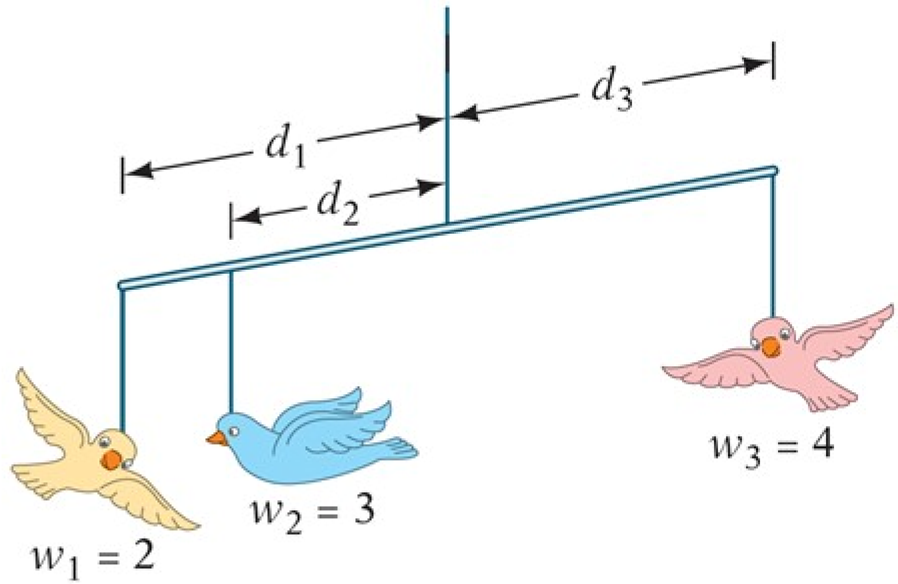
Application problems

Application problems

Example 3 is an application problem that requires more than two variables. The solution of this problem illustrates how the substitution method can be used to solve a system of linear equations in three variables.

Example 3

An artist is creating a mobile in which three objects will be suspended from a light rod that is 18 in. long, as shown at the right. The weight, in ounces, of each object is shown in the diagram.



For the mobile to balance, the objects must be positioned so that $w_1d_1 + w_2d_2 = w_3d_3$. The artist wants d_1 to be 1.5 times d_2. Find the distances d_1, d_2, and d_3 such that the mobile will balance.

Example 3

cont'd

Strategy:

There are three unknowns for this problem.

Use the information in the problem to write three equations with d_1, d_2, and d_3 as the variables.

The length of the rod is 18 in. Therefore, $d_1 + d_3 = 18$.
Using $w_1d_1 + w_2d_2 = w_3d_3$, we have $2d_1 + 3d_2 = 4d_3$.

The artist wants d_1 to be 1.5 times d_2. Thus $d_1 = 1.5d_2$.

Example 3 – *Solution*

(1) $$d_1 + d_3 = 18$$

Write a system of equations.

(2) $$2d_1 + 3d_2 = 4d_3$$

(3) $$d_1 = 1.5d_2$$

(4) $$1.5d_2 + d_3 = 18$$

(5) $$2(1.5d_2) + 3d_2 = 4d_3$$

To solve by substitution, first use equation (3) to replace d_1 in equation (1) and in equation (2).

$$6d_2 = 4d_3$$

Simplify equation (5).

(6) $$1.5d_2 = d_3$$

Divide each side of equation (5) by 4.

Example 3 – *Solution*

cont'd

$$1.5d_2 + 1.5d_2 = 18$$

Replace d_3 by $1.5d_2$ in equation (4) and solve for d_2.

$$3d_2 = 18$$

$$d_2 = 6$$

From equation (6), $d_3 = 1.5d_2 = 1.5(6) = 9$.

Substituting the value of d_3 into equation (1), we have $d_1 = 9$.

The distances are $d_1 = 9$ in., $d_2 = 6$ in., and $d_3 = 9$ in.