CHAPTER 4

Systems of Equations and Inequalities

4.2 Solving Systems of Linear Equations by the Addition Method

Objectives

1 Solve a system of two linear equations in two variables by the addition method

2 Solve a system of three linear equations in three variables by the addition method

Solve a system of two linear equations in two variables by the addition method

The **addition method** is an alternative method for solving a system of equations. This method is based on the Addition Property of Equations.

Use the addition method when it is not convenient to solve one equation for one variable in terms of another variable.

Sometimes adding the two equations does not eliminate one of the variables.

In this case, use the Multiplication Property of Equations to rewrite one or both of the equations so that when the equations are added, one of the variables is eliminated.

To do this, first choose which variable to eliminate. The coefficients of that variable must be additive inverses.

Multiply each equation by a constant that will produce coefficients that are additive inverses.

Example 1

Solve by the addition method.

A. $3x - 2y = 2x + 5$
$2x + 3y = -4$

B. $4x - 8y = 36$
$3x - 6y = 27$

Solution:

A. (1) $3x - 2y = 2x + 5$

(2) $2x + 3y = -4$

$x - 2y = 5$

$2x + 3y = -4$

Write equation (1) in the form $Ax + By = C$.

Example 1 – *Solution*

cont'd

$$-2(x - 2y) = -2(5)$$

$$2x + 3y = -4$$

To eliminate x, multiply each side of equation (1) by -2.

$$-2x + 4y = -10$$

Simplify.

$$\begin{array}{r} 2x + 3y = -4 \\ \hline 7y = -14 \end{array}$$

Add the equations.

$$y = -2$$

Example 1 – *Solution*

cont'd

$$2x + 3y = -4$$

Replace *y* in equation (2) by its value.

$$2x + 3(-2) = -4$$

Solve for *x*.

$$2x - 6 = -4$$

$$2x = 2$$

$$x = 1$$

The solution is (1, –2).

Example 1 – *Solution*

cont'd

B. (1) $4x - 8y = 36$

(2) $3x - 6y = 27$

$$3(4x - 8y) = 3(36)$$
$$-4(3x - 6y) = -4(27)$$

To eliminate x, multiply each side of equation (1) by 3 and each side of equation (2) by -4.

$$12x - 24y = 108$$

Simplify.

$$\begin{array}{r} -12x + 24y = -108 \\ \hline 0 = 0 \end{array}$$

Add the equations.

Example 1 – *Solution*

cont'd

0 = 0 is a true equation. The system of equations is dependent.

$$4x - 8y = 36 \qquad \text{Solve equation (1) for } y.$$

$$-8y = -4x + 36$$

$$y = \tfrac{1}{2}x - \tfrac{9}{2}$$

The solutions are the ordered pairs $\left(x, \frac{1}{2}x - \frac{9}{2}\right)$.

Solve a system of three linear equations in three variables by the addition method

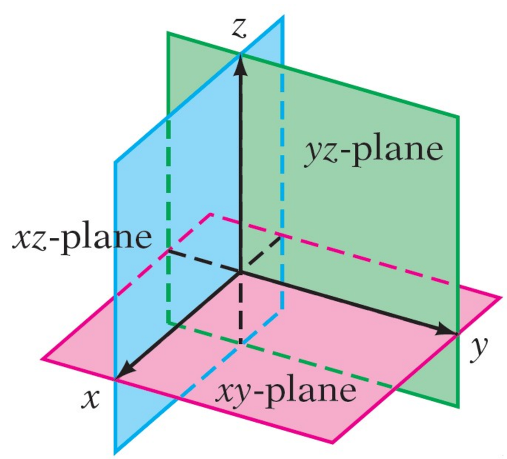
An equation of the form $Ax + By + Cz = D$, where A, B, and C are coefficients and D is a constant, is a **linear equation in three variables.** Two examples of this type of equation are shown below.

$2x + 4y - 3z = 7$ $\qquad\qquad$ $x - 6y + z = -3$

Graphing an equation in three variables requires a third coordinate axis perpendicular to the *xy*-plane. The third axis is commonly called the *z*-axis.

The result is a three-dimensional coordinate system called the ***xyz*-coordinate system.**

To help visualize a three-dimensional coordinate system, think of a corner of a room: the floor is the *xy*-plane, one wall is the *yz*-plane, and the other wall is the *xz*-plane. A three-dimensional coordinate system is shown below.

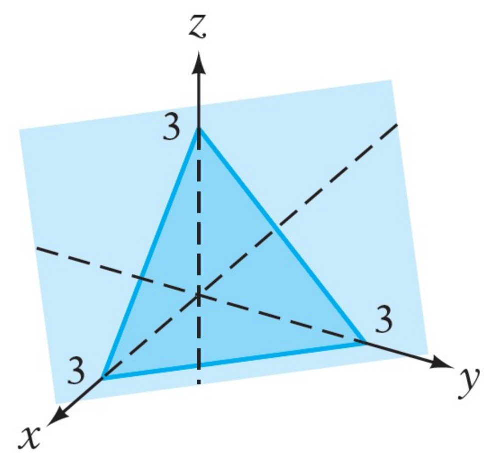


Each point in an *xyz*-coordinate system is the graph of an **ordered triple** (x, y, z).

Graphing an ordered triple requires three moves, the first along the x-axis, the second parallel to the y-axis, and the third parallel to the z-axis.

The graph of a linear equation in three variables is a plane. That is, if all the solutions of a linear equation in three variables were plotted in an xyz-coordinate system, the graph would look like a large piece of paper extending infinitely.

The graph of $x + y + z = 3$ is shown at the right.



Solve a system of three linear equations in three variables by the addition method

Just as a solution of an equation in two variables is an ordered pair (x, y), a **solution of an equation in three variables** is an ordered triple (x, y, z).

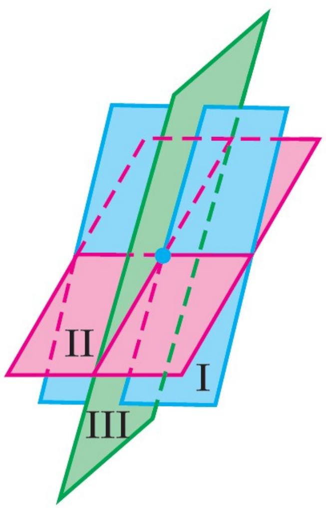
A **system of linear equations in three variables** is shown at the right. A **solution of a system of equations in three variables** is an ordered triple that is a solution of each equation of the system.

$$\begin{aligned} x - 2y + z &= 7 \\ 3x + y - 2z &= 4 \\ 2x - 3y + 5z &= 19 \end{aligned}$$

For a system of three equations in three variables to have a solution, the graphs of the equations must be three planes that intersect at a single point, be three planes that intersect along a common line, or all be the same plane. These situations are shown in the figures that follow.

The three planes shown in Figure A at the right intersect at a point.

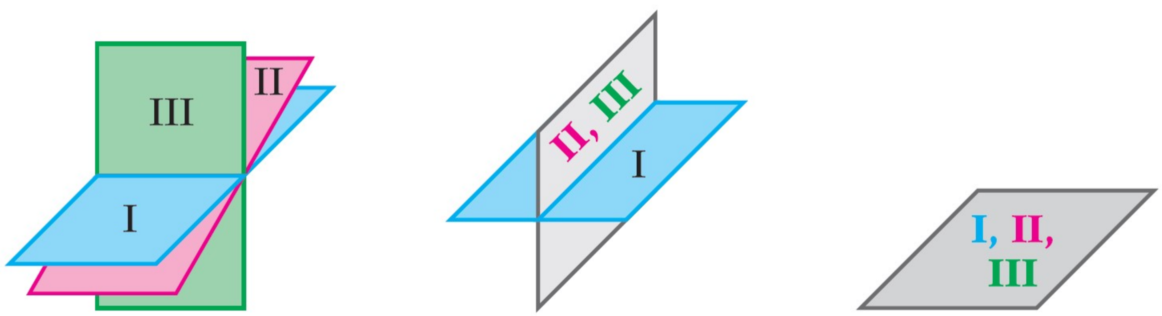
A system of equations represented by planes that intersect at a point is independent.



A

Graph of an Independent System of Equations

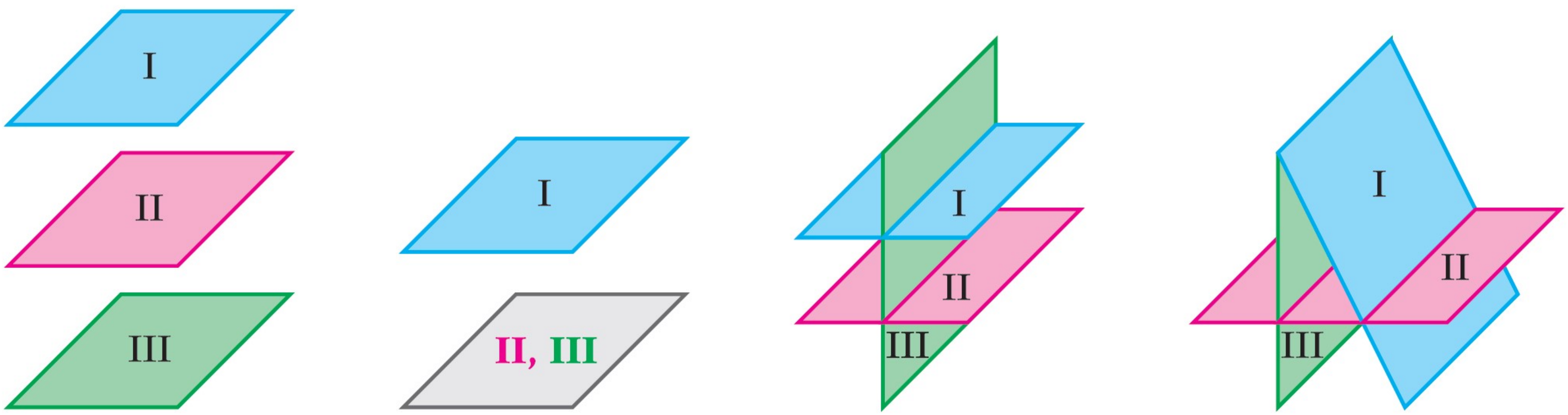
The three planes shown in Figures B and C below intersect along a common line. In Figure D, the three planes are all the same plane. The systems of equations represented by the planes in Figures B, C, and D are dependent systems.



B C D

Graphs of Dependent Systems of Equations

The systems of equations represented by the planes in the four figures below are inconsistent systems.



Graphs of Inconsistent Systems of Equations

A system of linear equations in three variables can be solved by using the addition method.

First, eliminate one variable from any two of the given equations. Then eliminate the same variable from any other two equations. The result will be a system of two equations in two variables.

Solve this system by the addition method.

Example 2

Solve:

$$\begin{aligned} 3x - y + 2z &= 1 \\ 2x + 3y + 3z &= 4 \\ x + y - 4z &= -9 \end{aligned}$$

Solution:

$$\begin{aligned} &(1) & 3x - y + 2z &= 1 \\ &(2) & 2x + 3y + 3z &= 4 \\ &(3) & x + y - 4z &= -9 \end{aligned}$$

Number the equations (1), (2), and (3).

Example 2 – *Solution*

cont'd

$$\begin{array}{r} 3x - y + 2z = 1 \\ x + y - 4z = -9 \\ \hline 4x - 2z = -8 \end{array}$$

Eliminate *y*. Add equations (1) and (3).

(4) $$2x - z = -4$$

Simplify the resulting equation by multiplying each side of the equation by $\frac{1}{2}$.

$$\begin{array}{r} 9x - 3y + 6z = 3 \\ 2x + 3y + 3z = 4 \\ \hline 11x + 9z = 7 \end{array}$$

(5)

Multiply equation (1) by 3 and add to equation (2).

Example 2 – *Solution*

cont'd

$$(4) \qquad 2x - z = -4$$

$$(5) \qquad 11x + 9z = 7$$

Solve the system of two equations, equations (4) and (5).

$$\begin{array}{r} 18x - 9z = -36 \\ 11x + 9z = 7 \\ \hline 29x = -29 \end{array}$$

$$x = -1$$

Multiply equation (4) by 9 and add to equation (5). Solve for *x*.

Example 2 – *Solution*

cont'd

$$
\begin{aligned}
(4) \qquad 2x - z &= -4 \\
2(-1) - z &= -4 \\
-2 - z &= -4 \\
-z &= -2 \\
z &= 2
\end{aligned}
$$

Replace *x* by −1 in equation (4). Solve for *z*.

Example 2 – *Solution*

cont'd

$$
\begin{aligned}
(3) \qquad x + y - 4z &= -9 \\
-1 + y - 4(2) &= -9 \\
-1 + y - 8 &= -9 \\
-9 + y &= -9 \\
y &= 0
\end{aligned}
$$

Replace *x* by –1 and *z* by 2 in equation (1), (2), or (3). Equation (3) is used here. Solve for *y*.

The solution is (–1, 0, 2).