CHAPTER

3

Linear Functions and Inequalities in Two Variables

3.6 Parallel and Perpendicular Lines

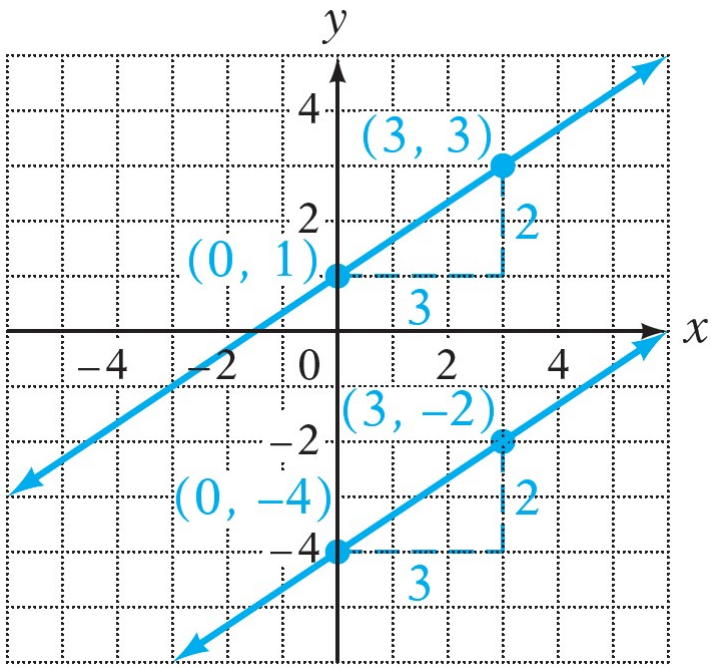
Objective

1. Find equations of parallel and perpendicular lines

Find equations of parallel and perpendicular lines

Find equations of parallel and perpendicular lines

Two lines that have the same slope and different *y*-intercepts do not intersect and are called **parallel lines**. The slope of each of the lines at the right is $\frac{2}{3}$. The lines are parallel.



Find equations of parallel and perpendicular lines

SLOPES OF PARALLEL LINES

Two nonvertical lines with slopes of m_1 and m_2 are parallel if and only if $m_1 = m_2$. Vertical lines are parallel lines.

EXAMPLES

1. The slopes of the graphs of $y = -\frac{2}{3}x + 2$ and $y = -\frac{2}{3}x - 3$ are both $-\frac{2}{3}$. The lines are parallel.
2. The graphs of $x = 2$ and $x = 5$ are vertical lines. The lines are parallel.
3. The slope of the graph of $y = 2x + 3$ is 2. The slope of the graph of $y = -2x + 1$ is -2. The slopes are not equal. The lines are not parallel.

Example 1

Are the graphs of $y = -\frac{3}{2}x - 5$ and $2x + 3y = 6$ parallel?

Solution:

Write, if necessary, each equation in the form $y = mx + b$, and then compare the values of m, the slopes of the lines. If the values of m are equal, the graphs are parallel.

The equation $y = -\frac{3}{2}x - 5$ is in the form $y = mx + b$.

The slope is $-\frac{3}{2}$.

Example 1 – *Solution*

cont'd

$$2x + 3y = 6$$

Write the equation $2x + 3y = 6$ in the form $y = mx + b$.

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

The slope is $-\frac{2}{3}$.

$$-\frac{2}{3} \neq -\frac{3}{2}$$

Compare the slopes.

Because the slopes are not the same, the graphs are not parallel.

Example 2

Find the equation of the line that contains the point $P(3, -1)$ and is parallel to the graph of $3x - 2y = 4$.

Solution:

$$3x - 2y = 4$$

Solve the equation for y to find the slope.

$$-2y = -3x + 4$$

$$y = \frac{3}{2}x - 2$$

$$m = \frac{3}{2}$$

The parallel line has the same slope as the given line.

Example 2 – *Solution*

cont'd

$$y - y_1 = m(x - x_1)$$

Use the point-slope formula.

$$y - (-1) = \frac{3}{2}(x - 3)$$

$m = \frac{3}{2}$, $(x_1, y_1) = (3, -1)$

$$y + 1 = \frac{3}{2}x - \frac{9}{2}$$

Solve for y.

$$y = \frac{3}{2}x - \frac{11}{2}$$

The equation of the line is $y = \frac{3}{2}x - \frac{11}{2}$.

Find equations of parallel and perpendicular lines

Two lines that intersect at right angles, as in Figure 1 below, are **perpendicular lines**. A horizontal line is perpendicular to a vertical line, as shown in Figure 2 below.

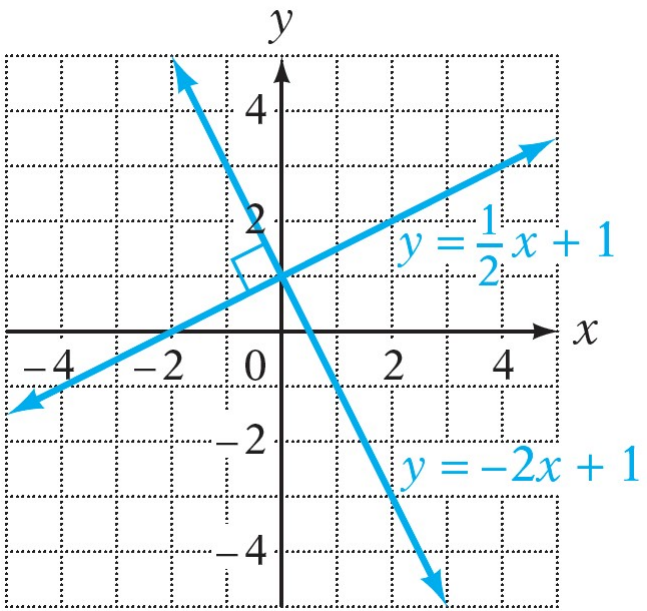


Figure 1

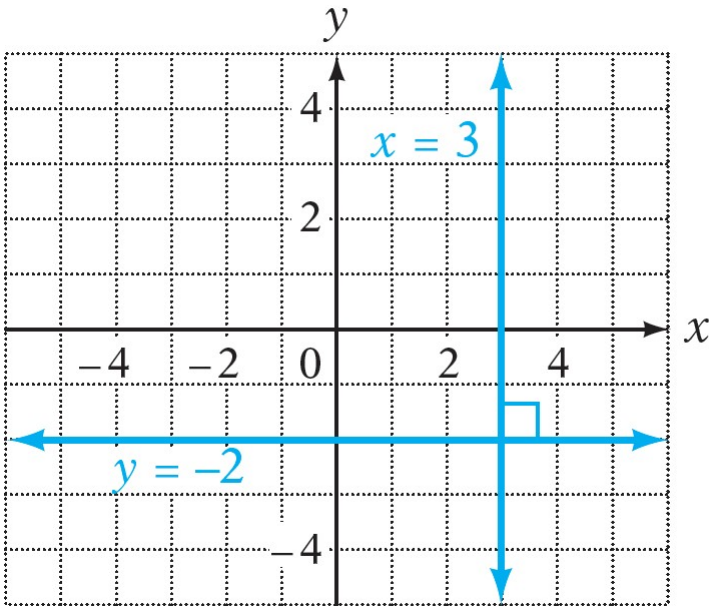


Figure 2

Find equations of parallel and perpendicular lines

SLOPES OF PERPENDICULAR LINES

If m_1 and m_2 are the slopes of two lines, neither of which is vertical, then the lines are perpendicular if and only if $m_1 \cdot m_2 = -1$. A vertical line is perpendicular to a horizontal line.

EXAMPLES

1. The slope of the graph of $y = \frac{3}{4}x + 1$ is $\frac{3}{4}$, and the slope of the graph of $y = -\frac{4}{3}x - 3$ is $-\frac{4}{3}$. The product of the two slopes is $\frac{3}{4} \cdot \left(-\frac{4}{3}\right) = -1$. The lines are perpendicular.

2. The graph of $x = -1$ is a vertical line, and the graph of $y = 5$ is a horizontal line. The lines are perpendicular.

3. The slope of the graph of $y = \frac{1}{2}x + 3$ is $\frac{1}{2}$. The slope of the graph of $y = 2x + 1$ is 2. The product of the slopes is $\frac{1}{2} \cdot 2 = 1 \neq -1$. The lines are not perpendicular.

Example 3

Is the line that contains the points with coordinates (–2, 3) and (–2, –5) perpendicular to the line that contains the points with coordinates (–1, 4) and (2, 4)?

Solution:

$$m_1 = \frac{-5 - 3}{-2 - (-2)}$$

$$= \frac{-8}{0}$$ undefined

Find the slope of the line between the points with coordinates $(-2, 3)$ and $(-2, -5)$. The slope is undefined. The line is vertical.

Example 3 – *Solution*

$$m_2 = \frac{4 - 4}{2 - (-1)}$$

$$= \frac{0}{3}$$

$$= 0$$

Find the slope of the line between the points with coordinates $(-1, 4)$ and $(2, 4)$. The slope is zero. The line is horizontal.

One line is vertical and one is horizontal. The lines are perpendicular.

Find equations of parallel and perpendicular lines

Solving $m_1 \cdot m_2 = -1$ for m_1 gives $m_1 = -\frac{1}{m_2}$.

This last equation states that the slopes of perpendicular lines are **negative reciprocals** of each other.

Example 4

Find the equation of the line that contains the point $P(3, -5)$ and is perpendicular to the graph of $y = -3x + 2$.

Solution:

The slope of $y = -3x + 2$ is -3. The slope of a line perpendicular to the given line is $\frac{1}{3}$, the negative reciprocal of -3.

$$y - y_1 = m(x - x_1)$$ **Use the point-slope formula.**

$$y - (-5) = \frac{1}{3}(x - 3)$$ **Substitute $m = \frac{1}{3}$ and $(x_1, y_1) = (3, -5)$.**

Example 4 – *Solution*

cont'd

$$y + 5 = \frac{1}{3}x - 1$$

Solve for y.

$$y = \frac{1}{3}x - 6$$

The equation of the line is $y = \frac{1}{3}x - 6$.