CHAPTER

3

# Linear Functions and Inequalities in Two Variables

# 3.3 Linear Functions

# Objectives

1 Graph a linear function

2 Graph an equation of the form $Ax + By = C$

3 Application problems

# Graph a linear function

# Graph a linear function

The ordered pairs of a function can be written as ($x$, $f$($x$)) or ($x$, $y$). The **graph of a function** is a graph of the ordered pairs ($x$, $y$) that belong to the function. Certain functions have characteristic graphs.

A function that can be written in the form $f(x) = mx + b$ (or $y = mx + b$) is called a **linear function** because its graph is a straight line.

Whether an equation is written as $f(x) = mx + b$ or as $y = mx + b$, the equation represents a linear function, and the graph of the equation is a straight line.

# Graph a linear function

Because the graph of a linear function is a straight line, and a straight line is determined by two points, the graph of a linear function can be drawn by finding only two of the ordered pairs of the function.

However, it is recommended that you find at least *three* ordered pairs to ensure accuracy.

# Example 1

Graph: $f(x) = -\frac{3}{2}x - 3$

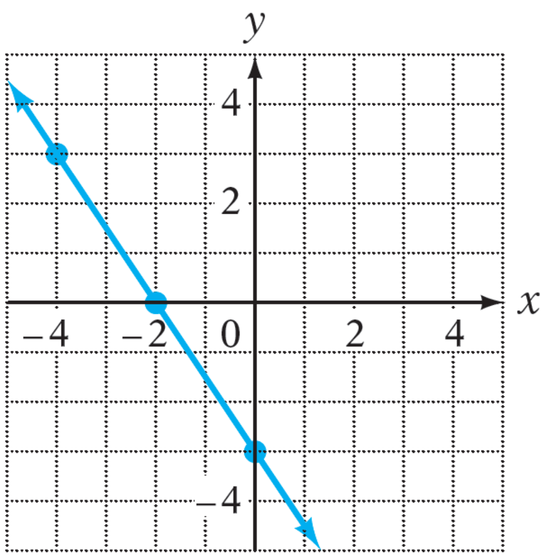
## Solution:

| $x$ | $y = f(x) = -\frac{3}{2}x - 3$ |
|---|---|
| $0$ | $-3$ |
| $-2$ | $0$ |
| $-4$ | $3$ |

**Find at least three ordered pairs. Because the coefficient of *x* is a fraction with denominator 2, choosing values of *x* that are divisible by 2 simplifies the evaluations. The ordered pairs can be displayed in a table.**

# Example 1 – *Solution*

cont'd




Graph the ordered pairs and draw a line through the points.

## Graph an equation of the form $Ax + By = C$

# Graph an equation of the form $Ax + By = C$

A **literal equation** is an equation with more than one variable. Examples of literal equations are $P = 2L + 2W$, $V = LWH$, $d = rt$, and $3x + 2y = 6$.

Linear equations of the form $y = mx + b$ are literal equations. In some cases, a linear equation has the form $Ax + By = C$. In such a case, it may be convenient to solve the equation for $y$ to write the equation in the form $y = mx + b$.

To solve for $y$, we use the same rules and procedures that we use to solve equations with numerical values.

# Graph an equation of the form $Ax + By = C$

We will show two methods of graphing an equation of the form $Ax + By = C$. In the first method, we solve the equation for $y$ and then follow the same procedure used for graphing an equation of the form $y = mx + b$.

# Example 2

Graph: $3x + 2y = 6$

Solution:

$$3x + 2y = 6$$

$$2y = -3x + 6$$ Solve the equation for $y$.

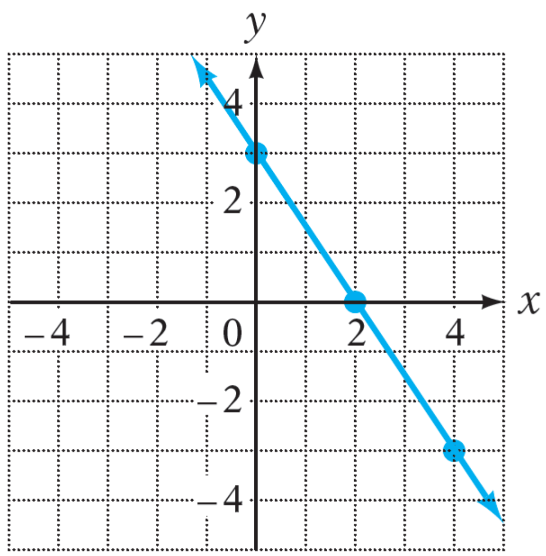
$$y = -\frac{3}{2}x + 3$$

| $x$ | $y = -\frac{3}{2}x + 3$ |
|---|---|
| 0 | 3 |
| 2 | 0 |
| 4 | $-3$ |

Find at least three solutions. Choose multiples of 2 for $x$.

# Example 2 – *Solution*

cont'd




**Graph the ordered pairs and draw a straight line through the points.**

# Graph an equation of the form $Ax + By = C$

An equation in which one of the variables is missing has a graph that is either a horizontal or a vertical line.

**DEFINITION OF A CONSTANT FUNCTION**

A function given by $f(x) = b$, where $b$ is a constant, is a **constant function.** The graph of a constant function is a horizontal line passing through $(0, b)$.

For the equation $x = 2$, the coefficient of $y$ is zero. For instance, the equation $x = 2$ can be written

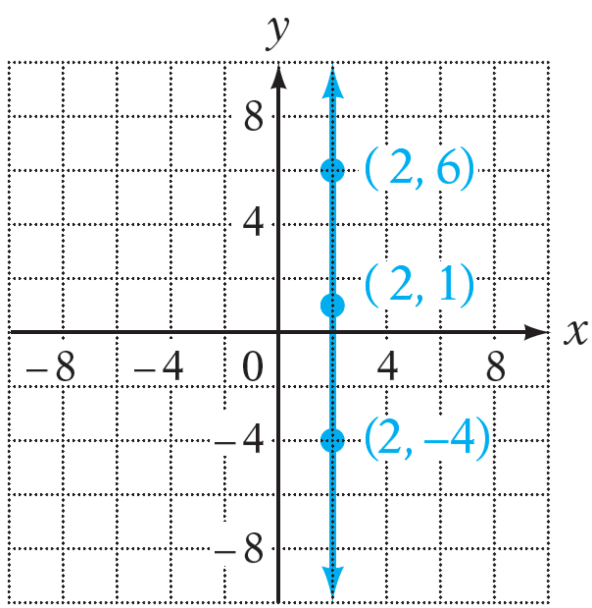
$$x + 0 \cdot y = 2$$

No matter what value of $y$ is chosen, $0 \cdot y = 0$, and therefore $x$ is always 2.

# Graph an equation of the form $Ax + By = C$

Some of the possible ordered-pair solutions are given in the table. The graph is shown below.

| $x$ | $y$ |
|---|---|
| 2 | 6 |
| 2 | 1 |
| 2 | −4 |



# Graph an equation of the form *Ax* + *By* = *C*

**GRAPH OF $x = a$**

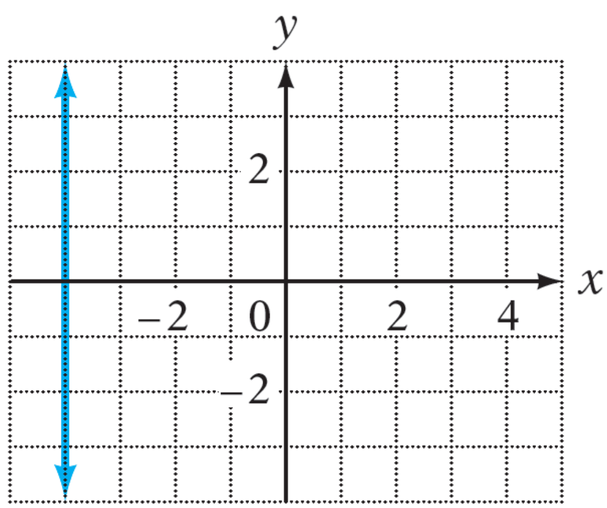
The graph of $x = a$ is a vertical line passing through the point $(a, 0)$.

We have known that a function is a set of ordered pairs in which no two ordered pairs have the same first coordinate. Because (2, 6), (2, 1), and (2, –4) are ordered-pair solutions of the equation $x = 2$, this equation does not represent a function, and its graph is not the graph of a function.

# Example 3

Graph: $x = -4$

Solution:

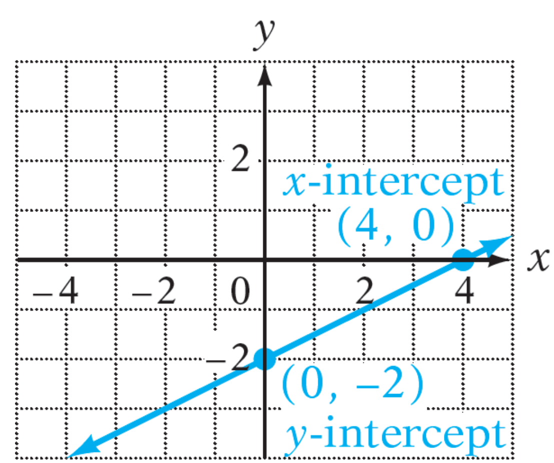



The graph of an equation of the form $x = a$ is a vertical line passing through the point whose coordinates are $(a, 0)$. The graph of $x = -4$ passes through $(-4, 0)$.

# Graph an equation of the form $Ax + By = C$

A second method of graphing straight lines uses the *intercepts* of the graph.

The graph of the equation $x – 2y = 4$ is shown at the right. The graph crosses the $x$-axis at the point with coordinates (4, 0). This point is called the ***x*-intercept**. The graph crosses the $y$-axis at the point with coordinates (0, –2). This point is called the ***y*-intercept**.




A linear equation can be graphed by finding the $x$- and $y$-intercepts and then drawing a line through the two points.

# Example 4

Graph $4x - y = 4$ by using the *x*- and *y*-intercepts.

Solution:

$x$-intercept: $4x - y = 4$

$$4x - 0 = 4$$ To find the *x*-intercept, let $y = 0$.

$$4x = 4$$

$$x = 1$$

The coordinates of the *x*-intercept are (1, 0).

# Example 4 – *Solution*

cont'd

$y$-intercept: $4x - y = 4$

$4(0) - y = 4$ To find the $y$-intercept, let $x = 0$.

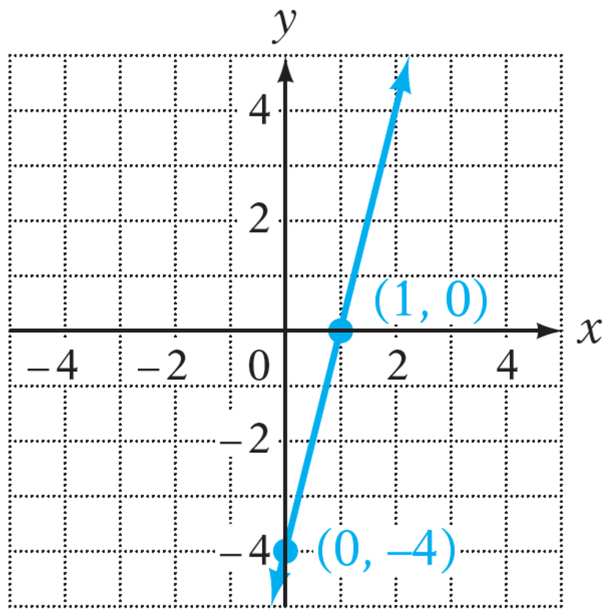
$-y = 4$

$y = -4$

The coordinates of the *y*-intercept are (0, –4).

# Example 4 – *Solution*

cont'd




**Graph the *x*- and *y*-intercepts. Draw a line through the two points.**

# Graph an equation of the form $Ax + By = C$

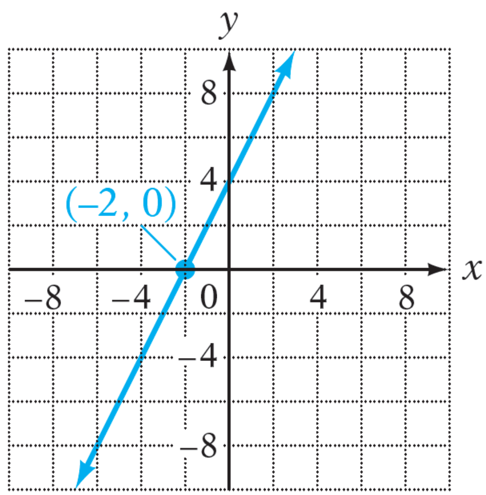
The graph of $f(x) = 2x + 4$ is shown at the right.

Evaluating the function when $x = -2$, we have

$$f(x) = 2x + 4$$

$$f(-2) = 2(-2) + 4 = -4 + 4$$

$$f(-2) = 0$$




$-2$ is the $x$ value for which $f(x) = 0$. A value of $x$ for which $f(x) = 0$ is called a *zero* of $f$.

# Graph an equation of the form $Ax + By = C$

Note that the *x*-intercept of the graph has coordinates (–2, 0). The *x*-coordinate of the *x*-intercept is –2, the zero of the function.

**ZERO OF A FUNCTION**

A value of $x$ for which $f(x) = 0$ is called a **zero** of $f$.

**EXAMPLES**

**1.** Let $f(x) = 3x - 6$ and $x = 2$.

$$f(2) = 3(2) - 6 = 0$$

Because $f(2) = 0$, 2 is a zero of $f$.

# Graph an equation of the form $Ax + By = C$

**2.** Let $g(x) = 4x + 8$ and $x = -2$.

$$g(-2) = 4(-2) + 8$$
$$= -8 + 8 = 0$$

Because $g(-2) = 0$, $-2$ is a zero of $g$.

**3.** Let $h(x) = 2x + 1$ and $x = 0$.

$$h(0) = 2(0) + 1$$
$$= 0 + 1$$
$$= 1 \neq 0$$

Because $h(0) \neq 0$, 0 is not a zero of $h$.

To find a zero of a function $f$, let $f(x) = 0$ and solve for $x$.

# Example 5

Find the zero of $f(x) = -2x - 3$.

Solution:

$$f(x) = -2x - 3$$

$$0 = -2x - 3$$ To find a zero of a function, let $f(x) = 0$.

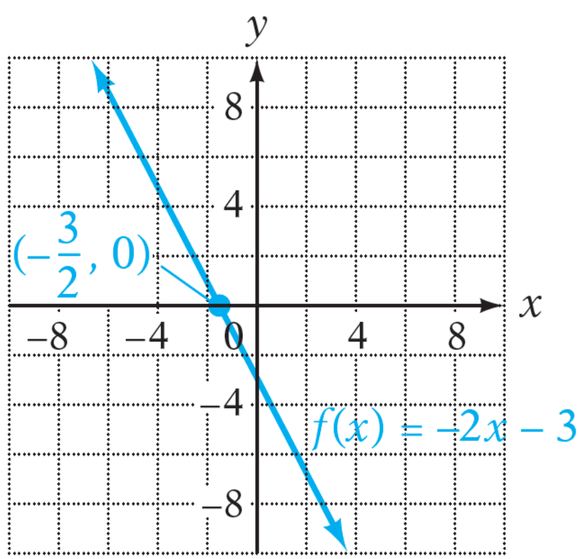
$$2x = -3$$ Solve for $x$.

$$x = -\frac{3}{2}$$

# Example 5 – *Solution*

cont'd

The zero is $-\frac{3}{2}$. The graph of *f* is shown below. Note that the *x*-coordinate of the *x*-intercept is the zero of *f* .



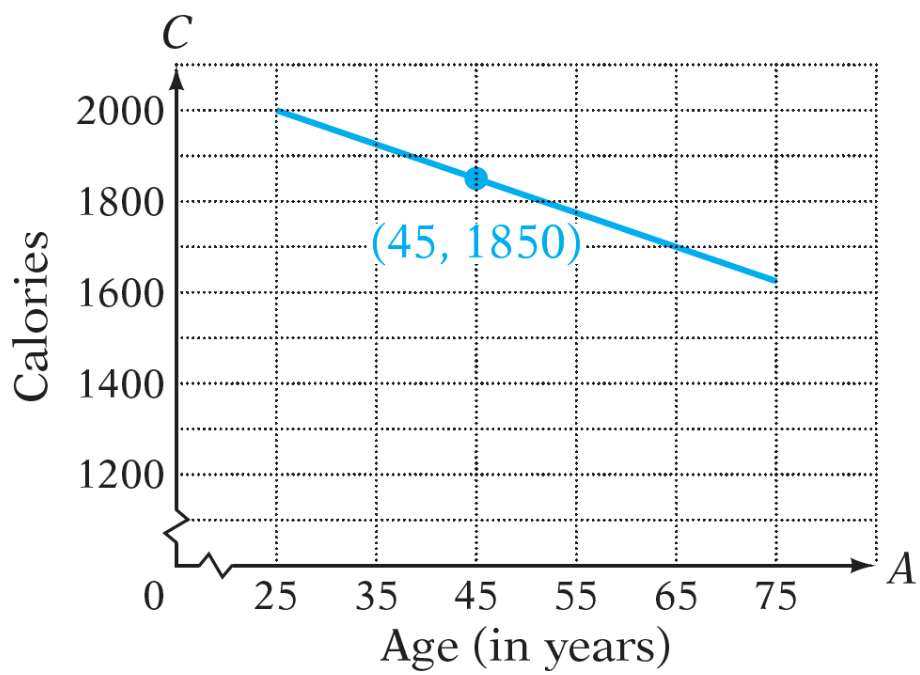
# Application problems

# Example 6

On the basis of data from *The Joy of Cooking*, the daily caloric allowance for a woman can be approximated by the equation $C = -7.5A + 2187.5$, where *C* is the caloric intake and *A* is the age of the woman. Graph this equation for $25 \le A \le 75$.

The point whose coordinates are (45, 1850) is on the graph. Write a sentence that describes the meaning of this ordered pair.

# Example 6 – *Solution*




The ordered pair (45, 1850) means that the caloric allowance for a 45-year-old woman is 1850 calories per day.