Solving Equations and Inequalities

CHAPTER 2

2.2 General Equations

Objectives

1. Solve equations of the form $ax + b = c$
2. Solve equations of the form $ax + b = cx + d$
3. Solve equations containing parentheses
4. Translate a sentence into an equation and solve

Solve equations of the form

$ax + b = c$

Solve equations of the form $ax + b = c$

In solving an equation of the form $ax + b = c$, the goal is to rewrite the equation in the form *variable* = *constant*.

This requires applying both the Addition and the Multiplication Properties of Equations.

Example 1

Solve: $3x - 7 = -5$

Solution:

$$3x - 7 = -5$$

$$3x - 7 + 7 = -5 + 7$$ Add 7 to each side of the equation.

$$3x = 2$$ Simplify.

$$\frac{3x}{3} = \frac{2}{3}$$ Divide each side of the equation by 3.

Example 1 – *Solution*

cont'd

$$x = \frac{2}{3}$$

Simplify. Now the equation is in the form *variable* = *constant*.

The solution is $\frac{2}{3}$.

Write the solution.

Solve equations of the form $ax + b = c$

Multiplying an equation that contains fractions by the LCM of the denominators is called **clearing denominators**.

It is an alternative method of solving an equation that contains fractions.

Clearing denominators is a method of solving equations. The process applies only to equations, never to expressions.

Example 3

Solve: $\frac{2}{3} + \frac{1}{4}x = -\frac{1}{3}$

Solution:

$$\frac{2}{3} + \frac{1}{4}x = -\frac{1}{3}$$

The equation contains fractions. Find the LCM of the denominators.

$$12\left(\frac{2}{3} + \frac{1}{4}x\right) = 12\left(-\frac{1}{3}\right)$$

The LCM of 3 and 4 is 12. Multiply each side of the equation by 12.

$$12\left(\frac{2}{3}\right) + 12\left(\frac{1}{4}x\right) = -4$$

Use the Distributive Property to multiply the left side of the equation by 12.

$$8 + 3x = -4$$

The equation now contains no fractions.

Example 3 – *Solution*

cont'd

$$8 - 8 + 3x = -4 - 8$$

$$3x = -12$$

$$\frac{3x}{3} = \frac{-12}{3}$$

$$x = -4$$

The solution is –4.

Solve equations of the form

$ax + b = cx + d$

Solve equations of the form $ax + b = cx + d$

In solving an equation of the form $ax + b = cx + d$, the goal is to rewrite the equation in the form *variable* = *constant*.

Begin by rewriting the equation so that there is only one variable term in the equation. Then rewrite the equation so that there is only one constant term.

Example 5

Solve: $4x - 3 = 8x - 7$

Solution:

$$4x - 3 = 8x - 7$$

$$4x - 8x - 3 = 8x - 8x - 7$$

Subtract $8x$ from each side of the equation.

$$-4x - 3 = -7$$

Simplify. Now there is only one variable term in the equation.

$$-4x - 3 + 3 = -7 + 3$$

Add 3 to each side of the equation.

Example 5 – *Solution*

cont'd

$-4x = -4$ Simplify. Now there is only one constant term in the equation.

$$\frac{-4x}{-4} = \frac{-4}{-4}$$

Divide each side of the equation by –4.

$x = 1$ Simplify. Now the equation is in the form *variable* = *constant.*

The solution is 1. Write the solution.

Solve equations containing parentheses

Solve equations containing parentheses

When an equation contains parentheses, one of the steps in solving the equation requires the use of the Distributive Property.

The Distributive Property is used to remove parentheses from a variable expression.

$$a(b + c) = ab + ac$$

Example 6

Solve: $3x - 4(2 - x) = 3(x - 2) - 4$

Solution:

$$3x - 4(2 - x) = 3(x - 2) - 4$$

$$3x - 8 + 4x = 3x - 6 - 4$$ Use the Distributive Property to remove parentheses.

$$7x - 8 = 3x - 10$$ Simplify.

$$7x - 3x - 8 = 3x - 3x - 10$$ Subtract $3x$ from each side of the equation.

$$4x - 8 = -10$$

Example 6 – *Solution*

cont'd

$4x - 8 + 8 = -10 + 8$ Add 8 to each side of the equation.

$4x = -2$

$$\frac{4x}{4} = \frac{-2}{4}$$ Divide each side of the equation by 4.

$$x = -\frac{1}{2}$$ The equation is in the form *variable* = *constant.*

The solution is $-\frac{1}{2}$.

Translate a sentence into an equation and solve

Translate a sentence into an equation and solve

An equation states that two mathematical expressions are equal. Therefore, translating a sentence into an equation requires recognizing the words or phrases that mean “equals.”

Some of the phrases that mean “equals” are *is*, *is equal to*, *amounts to*, and *represents*.

Once the sentence is translated into an equation, the equation can be solved by rewriting the equation in the form *variable* = *constant*.

Example 8

Translate “two times the sum of a number and eight equals the sum of four times the number and six” into an equation and solve.

Solution:

the unknown number: n

Assign a variable to the unknown quantity.

two times the sum of a number and eight	equals	the sum of four times the number and six

Find two verbal expressions for the same value.

$$2(n + 8) = 4n + 6$$

Write a mathematical expression for each verbal expression. Write the equals sign.

Example 8 – *Solution*

cont'd

Solve the equation.

$$2n + 16 = 4n + 6$$

$$2n - 4n + 16 = 4n - 4n + 6$$

$$-2n + 16 = 6$$

$$-2n + 16 - 16 = 6 - 16$$

$$-2n = -10$$

$$\frac{-2n}{-2} = \frac{-10}{-2}$$

$$n = 5$$

The number is 5.

Translate a sentence into an equation and solve

Integers are the numbers ... , –4, –3, –2, –1, 0, 1, 2, 3, 4,....

An **even integer** is an integer that is divisible by 2. Examples of even integers are –8, 0, and 22.

An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are –17, 1, and 39.

Translate a sentence into an equation and solve

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown below. (Assume the variable n represents an integer.)

$$11, 12, 13$$

$$-8, -7, -6$$

$$n, n + 1, n + 2$$

Examples of **consecutive even integers** are shown below. (Assume the variable n represents an even integer.)

$$24, 26, 28$$

$$-10, -8, -6$$

$$n, n + 2, n + 4$$

Translate a sentence into an equation and solve

Examples of **consecutive odd integers** are shown below. (Assume the variable *n* represents an odd integer.)

$$19, 21, 23$$

$$-1, 1, 3$$

$$n, n + 2, n + 4$$

Example 9

Find three consecutive even integers such that three times the second integer is six more than the sum of the first and third integers.

Strategy:

- First even integer: n

  Second even integer: $n + 2$

  Third even integer: $n + 4$

- Three times the second integer equals six more than the sum of the first and third integers.

Example 9 – *Solution*

$3(n + 2) = n + (n + 4) + 6$ Write an equation.

$3n + 6 = 2n + 10$ Solve for *n*.

$n + 6 = 10$

$n = 4$ The first even integer is 4.

$n + 2 = 4 + 2 = 6$ Substitute the value of *n* into the variable expressions for the second and third integers.

$n + 4 = 4 + 4 = 8$

The three consecutive even integers are 4, 6, and 8.