CHAPTER

2

Solving Equations and Inequalities

2.1 Introduction to Equations

Objectives

1 Determine whether a given number is a solution of an equation

2 Solve equations of the form $x + a = b$

3 Solve equations of the form $ax = b$

4 Applications of percent

Determine whether a given number is a solution of an equation

Determine whether a given number is a solution of an equation

An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation.

5 is a solution of the equation $x + 8 = 13$.

7 is not a solution of the equation $x + 8 = 13$.

Example 1

Is –3 a solution of the equation $4x + 16 = x^2 – 5$?

Solution:

$4x + 16$	$= x^2 - 5$	
$4(-3) + 16$	$(-3)^2 - 5$	Replace the variable by the given number, –3.
$-12 + 16$	$9 - 5$	Evaluate the numerical expressions using the Order of Operations Agreement.
4	$= 4$	Compare the results. If the results are equal, the given number is a solution. If the results are not equal, the given number is not a solution.

Yes, –3 is a solution of the equation $4x + 16 = x^2 – 5$.

Solve equations of the form $x + a = b$

Solve equations of the form $x + a = b$

To **solve an equation** means to find a solution of the equation. The simplest equation to solve is an equation of the form **variable = constant**, because the constant is the solution.

ADDITION PROPERTY OF EQUATIONS

The same number or variable term can be added to each side of an equation without changing the solution of the equation.

Solve equations of the form $x + a = b$

In solving an equation, the goal is to rewrite the given equation in the form variable = constant.

The Addition Property of Equations can be used to rewrite an equation in this form.

The Addition Property of Equations is used to remove a term from one side of an equation by adding the opposite of that term to each side of the equation.

Example 3

Solve and check: $y - 6 = 9$

Solution:

$y - 6 = 9$	The goal is to rewrite the equation in the form *variable* = *constant*.
$y - 6 + 6 = 9 + 6$	Add 6 to each side of the equation (the Addition Property of Equations).
$y + 0 = 15$	Simplify using the Inverse Property of Addition.
$y = 15$	Simplify using the Addition Property of Zero. Now the equation is in the form *variable = constant.*

Example 3 – *Solution*

cont'd

Check:

$$\begin{array}{c|c} y - 6 & = 9 \\ \hline 15 - 6 & 9 \end{array}$$

$$9 = 9$$ This is a true equation. The solution checks.

The solution is 15. Write the solution.

Solve equations of the form $x + a = b$

Because subtraction is defined in terms of addition, the Addition Property of Equations makes it possible to subtract the same number from each side of an equation without changing the solution of the equation.

Example 4

Solve: $\frac{1}{2} = y + \frac{2}{3}$

Solution:

$$\frac{1}{2} = y + \frac{2}{3}$$

$$\frac{1}{2} - \frac{2}{3} = y + \frac{2}{3} - \frac{2}{3}$$ Subtract $\frac{2}{3}$ from each side of the equation.

$$-\frac{1}{6} = y$$ Simplify each side of the equation.

The solution is $-\frac{1}{6}$.

Solve equations of the form $x + a = b$

Note from the solution to Example 4 that an equation can be rewritten in the form *constant* = *variable*.

Whether the equation is written in the form *variable* = *constant* or in the form *constant* = *variable*, the solution is the constant.

Solve equations of the form

$ax = b$

Solve equations of the form *ax* = *b*

MULTIPLICATION PROPERTY OF EQUATIONS

Each side of an equation can be multiplied by the same nonzero number without changing the solution of the equation.

This property is used in solving equations.

Note the effect of multiplying each side of the equation $2x = 6$ by the reciprocal of the coefficient 2.

$$2x = 6$$

$$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 6$$

Solve equations of the form $ax = b$

After each side of the equation is simplified, the equation is in the form variable = constant.

$$1x = 3$$

$$x = 3$$

$$\boxed{\text{variable}} = \boxed{\text{constant}}$$

The solution is 3.

The solution is the constant.

Solve equations of the form *ax* = *b*

In solving an equation, the goal is to rewrite the given equation in the form variable = constant.

The Multiplication Property of Equations can be used to rewrite an equation in this form.

The Multiplication Property of Equations is used to write the variable term with a coefficient of 1 by multiplying each side of the equation by the reciprocal of the coefficient.

Example 5

Solve: $\frac{3x}{4} = -9$

Solution:

$$\frac{3x}{4} = -9$$

$\frac{3x}{4} = \frac{3}{4}x$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$

Multiply each side of the equation by the reciprocal of the coefficient $\frac{3}{4}$ (the Multiplication Property of Equations).

$$1x = -12$$

Simplify using the Inverse Property of Multiplication.

Example 5 – *Solution*

cont'd

$x = -12$	Simplify using the Multiplication Property of One. Now the equation is in the form *variable = constant.*
The solution is –12.	Write the solution.

Solve equations of the form *ax* = *b*

Because division is defined in terms of multiplication, the Multiplication Property of Equations makes it possible to divide each side of an equation by the same number without changing the solution of the equation.

Solve equations of the form $ax = b$

When using the Multiplication Property of Equations to solve an equation, multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction.

Divide each side of the equation by the coefficient when the coefficient is an integer or a decimal.

Applications of percent

Applications of percent

Solving a problem that involves a percent requires solving the basic percent equation.

THE BASIC PERCENT EQUATION

$$\text{Percent} \cdot \text{base} = \text{amount}$$
$$P \cdot B = A$$

To translate a problem involving a percent into an equation, remember that the word *of* translates into "multiply" and the word *is* translates into "=." The base usually follows the word *of*.

Example 9

During a recent year, nearly 1.2 million dogs or litters were registered with the American Kennel Club. The most popular breed was the Labrador retriever, with 172,841 registered. What percent of the registrations were Labrador retrievers? Round to the nearest tenth of a percent. (*Source:* American Kennel Club)

Strategy:

To find the percent, solve the basic percent equation using B = 1.2 million = 1,200,000 and A = 172,841. The percent is unknown.

Example 9 – *Solution*

$$PB = A$$

$$P(1{,}200{,}000) = 172{,}841$$

B = 1,200,000; A = 172,841.

$$\frac{P(1{,}200{,}000)}{1{,}200{,}000} = \frac{172{,}841}{1{,}200{,}000}$$

Divide each side of the equation by 1,200,000.

$$P \approx 0.144$$

$$P \approx 14.4\%$$

Rewrite the decimal as a percent.

Approximately 14.4% of the registrations were Labrador retrievers.

Applications of percent

The simple interest that an investment earns is given by the equation

$$\boldsymbol{I = Prt}$$

where I is the simple interest, P is the principal, or amount invested, r is the simple interest rate, and t is the time.

Example 11

In April, Marshall Wardell was charged \$8.72 in interest on an unpaid credit card balance of \$545. Find the annual interest rate for this credit card.

Strategy:

The interest is \$8.72. Therefore, $I = 8.72$. The unpaid balance is \$545. This is the principal on which interest is calculated. Therefore, $P = 545$. The time is 1 month. Because the *annual* interest rate must be found and the time is given as 1 month, write 1 month as $\frac{1}{12}$ year. Therefore, $t = \frac{1}{12}$. To find the interest rate, solve the equation $I = Prt$ for r.

Example 11 – *Solution*

$I = Prt$

$8.72 = 545r\left(\frac{1}{12}\right)$ $\quad$ $I = 8.72$, $P = 545$, $t = \frac{1}{12}$

$8.72 = \frac{545}{12}r$ $\quad$ Multiply 545 by $\frac{1}{12}$.

$\frac{12}{545}(8.72) = \frac{12}{545}\left(\frac{545}{12}r\right)$ $\quad$ Multiply each side of the equation by $\frac{12}{545}$, the reciprocal of $\frac{545}{12}$.

$0.192 = r$

The annual interest rate is 19.2%.

Applications of percent

Problems involving mixtures are solved using the percent mixture equation

$$Q = Ar$$

where Q is the quantity of a substance in the solution, A is the amount of the solution, and r is the percent concentration of the substance.

Example 12

To make a certain color of blue, 4 oz of cyan must be contained in 1 gal of paint. What is the percent concentration of cyan in the paint?

Strategy:

The amount of cyan is given in ounces and the amount of paint is given in gallons; we must convert ounces to gallons or gallons to ounces. For this problem, we will convert gallons to ounces: 1 gal = 128 oz. Therefore, $A = 128$. The quantity of cyan in the paint is 4 oz; $Q = 4$. To find the percent concentration, solve the equation $Q = Ar$ for r.

Example 12 – *Solution*

$$Q = Ar$$

$$4 = 128r$$

$$\frac{4}{128} = \frac{128r}{128}$$

$$0.03125 = r$$

The percent concentration of cyan is 3.125%.