Real Numbers and Variable Expressions

CHAPTER 1

1.5 Variable Expressions

Objectives

1 Evaluate variable expressions

2 The Properties of the Real Numbers

3 Simplify variable expressions using the Properties of Addition

4 Simplify variable expressions using the Properties of Multiplication

Objectives

5 Simplify variable expressions using the Distributive Property

6 Simplify general variable expressions

Evaluate variable expressions

Evaluate variable expressions

Often we discuss a quantity without knowing its exact value, such as the price of gold next month, the cost of a new automobile next year, or the tuition for next semester.

In algebra, a letter of the alphabet is used to stand for a quantity that is unknown or one that can change, or *vary.* The letter is called a **variable**.

An expression that contains one or more variables is called a **variable expression**.

Evaluate variable expressions

A variable expression is shown below.

$$3x^2 - 5y + 2xy - x - 7$$

The expression can be rewritten by writing subtraction as the addition of the opposite.

$$3x^2 + (-5y) + 2xy + (-x) + (-7)$$

Note that the expression has five addends.

Evaluate variable expressions

The **terms** of a variable expression are the addends of the expression. The expression has five terms.

5 terms

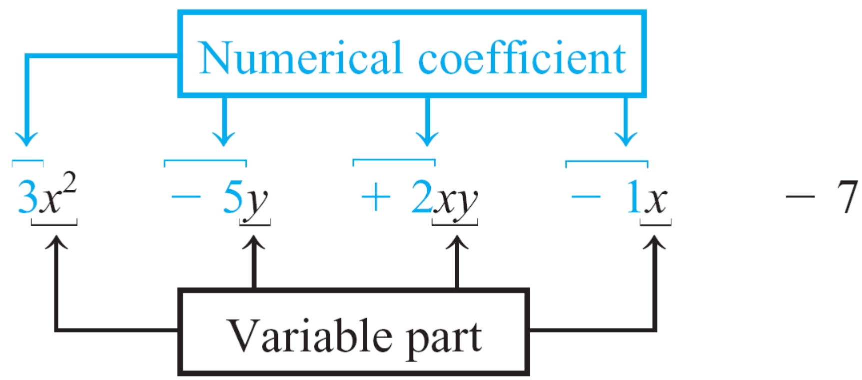
$$\underbrace{\overbrace{3x^2}\;\;\overbrace{-\,5y}\;\;\overbrace{+\,2xy}\;\;\overbrace{-\,x}}_{\text{Variable terms}}\qquad\underbrace{\overbrace{-\,7}}_{\text{Constant term}}$$

The terms $3x^2$, $–5y$, $2xy$, and $–x$ are **variable terms**.

The term –7 is a **constant term**, or simply a **constant**.

Evaluate variable expressions

Each variable term is composed of a **numerical coefficient** and a **variable part** (the variable or variables and their exponents).



When the numerical coefficient is 1 or –1, the 1 is usually not written ($x = 1x$ and $-x = -1x$).

Example 1

Name the variable terms of the expression $2a^2 - 5a + 7$.

Solution:

$2a^2$, $-5a$

Evaluate variable expressions

Replacing the variable or variables in a variable expression with numbers and then simplifying the resulting numerical expression is called **evaluating the variable expression.**

Example 2

Evaluate $ab - b^2$ when $a = 2$ and $b = -3$.

Solution:

$ab - b^2$

$2(-3) - (-3)^2$

Replace each variable in the expression with the number it represents.

$= 2(-3) - 9$

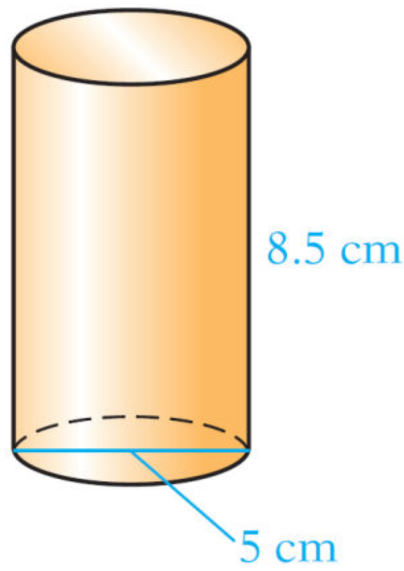
Use the Order of Operations Agreement to simplify the resulting numerical expression.

$= -6 - 9$

$= -15$

Example 5

The diameter of the base of a right circular cylinder is 5 cm. The height of the cylinder is 8.5 cm. Find the volume of the cylinder. Round to the nearest tenth.



Example 5 – *Solution*

$V = \pi r^2 h$

Use the formula for the volume of a right circular cylinder.

$V = \pi(2.5)^2(8.5)$

$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5$

$V = \pi(6.25)(8.5)$

Use the π key on your calculator to enter the value of π.

$V \approx 166.9$

The volume is approximately 166.9 cm^3.

The Properties of the Real Numbers

The Properties of the Real Numbers

The Properties of the Real Numbers describe the ways operations on numbers can be performed. Here are some of the Properties of the Real Numbers described algebraically and in words. An example of each is provided.

THE COMMUTATIVE PROPERTY OF ADDITION

If a and b are real numbers, then $a + b = b + a$.

Two terms can be added in either order; the sum is the same.

EXAMPLE

$4 + 3 = 7$ and $3 + 4 = 7$

The Properties of the Real Numbers

THE COMMUTATIVE PROPERTY OF MULTIPLICATION

If a and b are real numbers, then $a \cdot b = b \cdot a$.

Two factors can be multiplied in either order; the product is the same.

EXAMPLE

$(5)(-2) = -10$ and $(-2)(5) = -10$

THE ASSOCIATIVE PROPERTY OF ADDITION

If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

When three or more terms are added, the terms can be grouped (with parentheses, for example) in any order; the sum is the same.

EXAMPLE

$2 + (3 + 4) = 2 + 7 = 9$ and $(2 + 3) + 4 = 5 + 4 = 9$

The Properties of the Real Numbers

THE ASSOCIATIVE PROPERTY OF MULTIPLICATION

If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

When three or more factors are multiplied, the factors can be grouped in any order; the product is the same.

EXAMPLE

$(-3 \cdot 4) \cdot 5 = -12 \cdot 5 = -60$ and $-3 \cdot (4 \cdot 5) = -3 \cdot 20 = -60$

THE ADDITION PROPERTY OF ZERO

If a is a real number, then $a + 0 = a$ and $0 + a = a$.

The sum of a term and zero is the term.

EXAMPLE

$4 + 0 = 4$ and $0 + 4 = 4$

The Properties of the Real Numbers

THE MULTIPLICATION PROPERTY OF ZERO

If a is a real number, then $a \cdot 0 = 0$ and $0 \cdot a = 0$.

The product of a term and zero is zero.

EXAMPLE

$(5)0 = 0 \quad \text{and} \quad 0(5) = 0$

THE MULTIPLICATION PROPERTY OF ONE

If a is a real number, then $a \cdot 1 = a$ and $1 \cdot a = a$.

The product of a term and 1 is the term.

EXAMPLE

$6 \cdot 1 = 6 \quad \text{and} \quad 1 \cdot 6 = 6$

The Properties of the Real Numbers

THE INVERSE PROPERTY OF ADDITION

If a is a real number, then $a + (-a) = 0$ and $(-a) + a = 0$.

The sum of a number and its additive inverse (or opposite) is zero.

EXAMPLE

$8 + (-8) = 0 \quad \text{and} \quad (-8) + 8 = 0$

THE INVERSE PROPERTY OF MULTIPLICATION

If a is a real number and $a \neq 0$, then $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.

The product of a number and its reciprocal is 1.

EXAMPLE

$7 \cdot \frac{1}{7} = 1 \quad \text{and} \quad \frac{1}{7} \cdot 7 = 1$

The Properties of the Real Numbers

$\frac{1}{a}$ is the **reciprocal** of *a*. $\frac{1}{a}$ is also called the **multiplicative inverse** of *a*.

THE DISTRIBUTIVE PROPERTY

If a, b, and c are real numbers, then $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$.

By the Distributive Property, the term outside the parentheses is multiplied by each term inside the parentheses.

EXAMPLE

$$\begin{aligned} 2(3 + 4) &= 2 \cdot 3 + 2 \cdot 4 \\ 2 \cdot 7 &= 6 + 8 \\ 14 &= 14 \end{aligned} \qquad \begin{aligned} (4 + 5)2 &= 4 \cdot 2 + 5 \cdot 2 \\ (9)2 &= 8 + 10 \\ 18 &= 18 \end{aligned}$$

Example 6

Complete the statement by using the Commutative Property of Multiplication.

$(6)(5) = (?)(6)$

Solution:

$(6)(5) = (5)(6)$

The Commutative Property of Multiplication states that $a \cdot b = b \cdot a$.

Example 7

Identify the property that justifies the statement.

$2(8 + 5) = 16 + 10$

Solution:

The Distributive Property

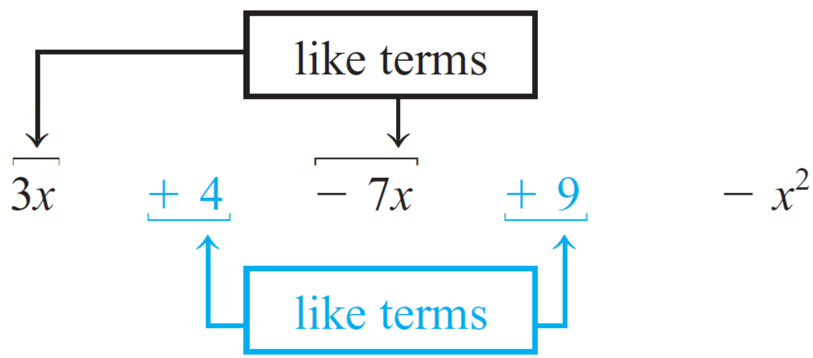
The Distributive Property states that $a(b + c) = ab + ac$.

Simplify variable expressions using the Properties of Addition

Simplify variable expressions using the Properties of Addition

Like terms of a variable expression are terms with the same variable parts. The terms $3x$ and $-7x$ are like terms.

Constant terms are like terms. 4 and 9 are like terms.



To **combine like terms,** use the Distributive Property $ba + ca = (b + c)a$ to add the coefficients.

Example 8

Simplify. **A.** $-2y + 3y$ **B.** $5x - 11x$

Solution:

A. $-2y + 3y$

$= (-2 + 3)y$ Use the Distributive Property $ba + ca = (b + c)a$.

$= 1y$ Add the coefficients.

$= y$ Use the Multiplication Property of One.

Example 8 – *Solution*

cont'd

B. $5x - 11x$

$= [5 + (-11)]x$ Use the Distributive Property $ba + ca = (b + c)a$.

$= -6x$ Add the coefficients.

Example 9

Simplify. **A.** $8x + 3y - 8x$ **B.** $4x^2 + 5x - 6x^2 - 2x$

Solution:

A. $8x + 3y - 8x$

$= 3y + 8x - 8x$ Use the Commutative Property of Addition to rearrange the terms.

$= 3y + (8x - 8x)$ Use the Associative Property of Addition to group like terms.

$= 3y + 0$ Use the Inverse Property of Addition.

$= 3y$ Use the Addition Property of Zero.

Example 9 – *Solution*

cont'd

B. $4x^2 + 5x - 6x^2 - 2x$

$= 4x^2 - 6x^2 + 5x - 2x$ — Use the Commutative Property of Addition to rearrange the terms.

$= (4x^2 - 6x^2) + (5x - 2x)$ — Use the Associative Property of Addition to group like terms.

$= -2x^2 + 3x$ — Combine like terms.

Simplify variable expressions using the Properties of Multiplication

Simplify variable expressions using the Properties of Multiplication

The Properties of Multiplication are used in simplifying variable expressions.

The Associative Property is used when multiplying three or more factors.

$$2(3x) = (2 \cdot 3)x = 6x$$

The Commutative Property can be used to change the order in which factors are multiplied.

$$(3x) \cdot 2 = 2 \cdot (3x) = 6x$$

Simplify variable expressions using the Properties of Multiplication

By the Multiplication Property of One, the product of a term and 1 is the term.

$$(8x)(1) = (1)(8x) = 8x$$

By the Inverse Property of Multiplication, the product of a term and its reciprocal is 1.

$$5x \cdot \frac{1}{5x} = \frac{1}{5x} \cdot 5x = 1, x \neq 0$$

Example 10

Simplify **A.** $2(-x)$ **B.** $\frac{3}{2}\left(\frac{2x}{3}\right)$ **C.** $(16x)2$

Solution:

A. $2(-x) = 2(-1 \cdot x)$ $-x = -1x = -1 \cdot x$

$= [2 \cdot (-1)]\, x$ Use the Associative Property of Multiplication to group factors.

$= -2x$ Multiply.

Example 10 – *Solution*

cont'd

B. $\dfrac{3}{2}\left(\dfrac{2x}{3}\right) = \dfrac{3}{2}\left(\dfrac{2}{3}x\right)$ Note that $\dfrac{2x}{3} = \dfrac{2}{3} \cdot \dfrac{x}{1} = \dfrac{2}{3}x$.

$= \left(\dfrac{3}{2} \cdot \dfrac{2}{3}\right)x$ Use the Associative Property of Multiplication to group factors.

$= 1x$ Use the Inverse property of Multiplication.

$= x$ Use the Multiplication property of One.

Example 10 – *Solution*

cont'd

C. $(16x)2 = 2(16x)$ Use the Commutative Property of Multiplication to rearrange factors.

$= (2 \cdot 16)x$ Use the Associative property of Multiplication to group factors.

$= 32x$ Multiply

Simplify variable expressions using the Distributive Property

Simplify variable expression using the Distributive Property

The Distributive Property is used to remove parentheses from a variable expression.

An extension of the Distributive Property is used when an expression contains more than two terms.

Example 11

Simplify. **A.** $-3(5 + x)$ **B.** $-(2x - 4)$

C. $(2y - 6)2$ **D.** $5(3x + 7y - z)$

Solution:

A. $-3(5 + x)$

$= -3(5) + (-3)x$ Use the Distributive Property.

$= -15 - 3x$ Multiply

Example 11 – *Solution*

cont'd

B. $-(2x - 4)$

$= -1(2x - 4)$ Just as $-x = -1x$, $-(2x - 4) = -1(2x - 4)$

$= -1(2x) - (-1)(4)$ Use the Distributive Property.

$= -2x + 4$

Note: When a negative sign immediately precedes the parentheses, remove the parentheses and change the sign of each term inside the parentheses.

Example 11 – *Solution*

cont'd

C. $(2y - 6)2$

$= (2y)(2) - (6)(2)$ Use the Distributive Property $(b + c)a = ba + ca$.

$= 4y - 12$

D. $5(3x + 7y - z)$

$= 5(3x) + 5(7y) - 5(z)$ Use the Distributive Property

$= 15x + 35y - 5z$

Simplify general variable expressions

Simplify general variable expressions

When simplifying variable expressions, use the Distributive Property to remove parentheses and brackets used as grouping symbols.

Example 12

Simplify: $4(x - y) - 2(-3x + 6y)$

Solution:

$4(x - y) - 2(-3x + 6y)$

$= 4x - 4y + 6x - 12y$ Use the Distributive Property to remove parentheses.

$= 10x - 16y$ Combine like terms.