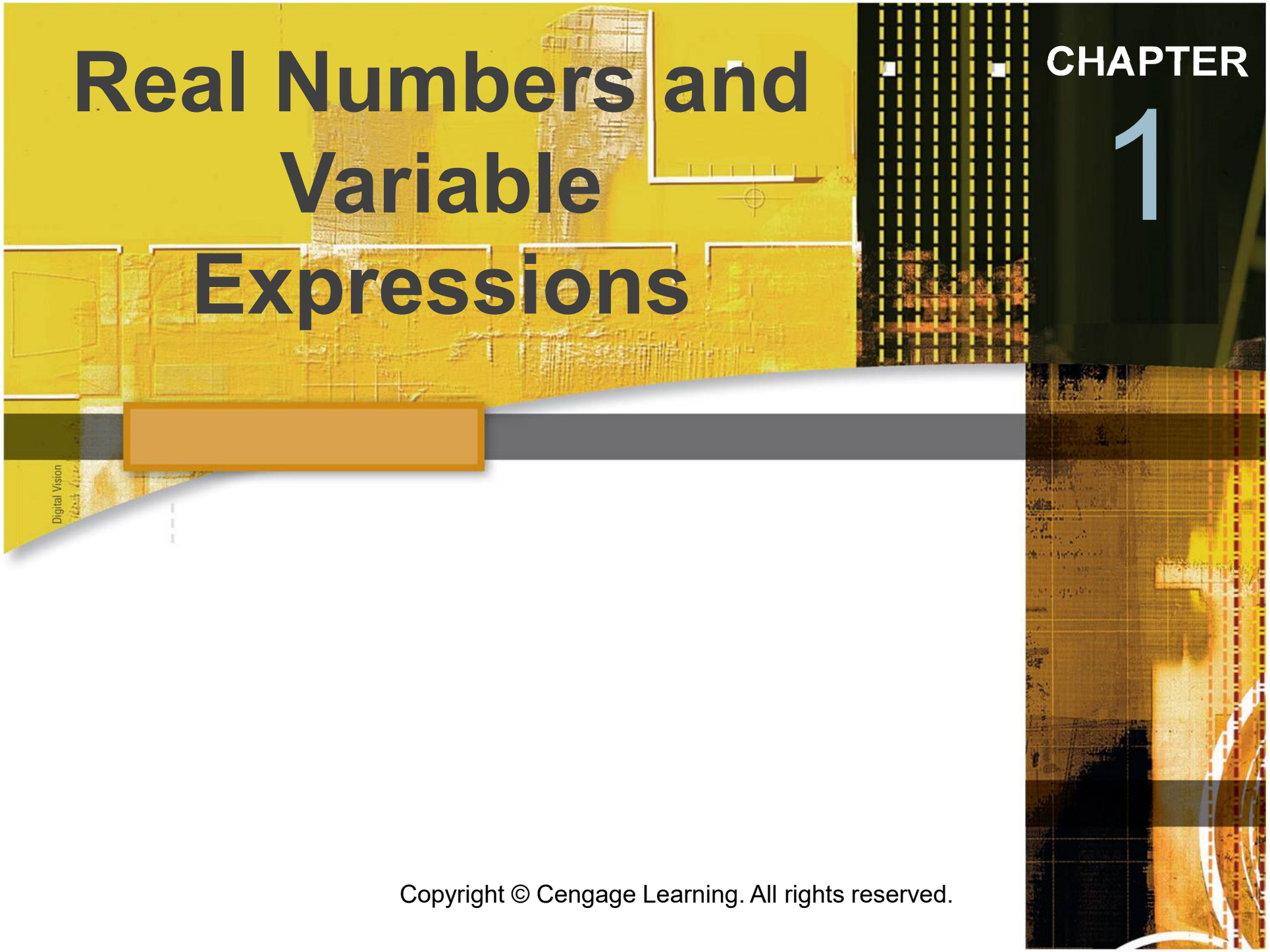
Real Numbers and Variable Expressions

CHAPTER 1



1.4 Exponents and the Order of Operations Agreement

Objectives

1. Exponential expressions
2. The Order of Operations Agreement

Exponential expressions

Exponential expressions

Repeated multiplication of the same factor can be written using an exponent.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ ← exponent, base

$a \cdot a \cdot a \cdot a = a^4$ ← exponent, base

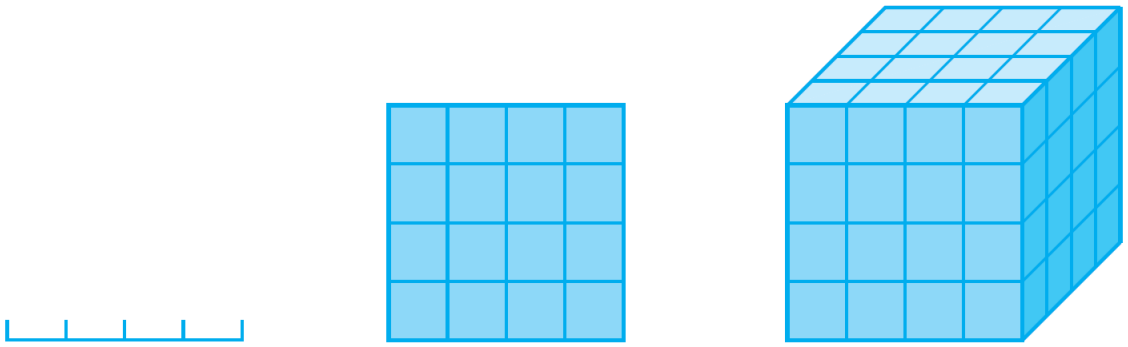
The **exponent** indicates how many times the factor, called the base, occurs in the multiplication.

The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in **factored form**.

The exponential expression 2^5 is in **exponential form**.

Exponential expressions

There is a geometric interpretation of the first three natural-number powers.



$4^1 = 4$
Length: 4 ft

$4^2 = 16$
Area: 16 ft^2

$4^3 = 64$
Volume: 64 ft^3

To evaluate an exponential expression, write each factor as many times as indicated by the exponent. Then multiply.

Example 1

Evaluate $(-4)^2$ and -4^2.

Solution:

$$(-4)^2 = (-4)(-4)$$
$$= 16$$

The –4 is squared only when the negative sign is *inside* the parentheses. In $(-4)^2$, we are squaring –4; in -4^2, we are finding the opposite of 4^2.

$$-4^2 = -(4 \cdot 4)$$
$$= -16$$

Example 3

Evaluate $(-3)^2 \cdot 2^3$ and $\left(-\frac{2}{3}\right)^3$.

Solution:

$$(-3)^2 \cdot 2^3 = (-3)(-3) \cdot (2)(2)(2)$$

$$= 9 \cdot 8$$

$$= 72$$

$$\left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = -\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3}$$

$$= -\frac{8}{27}$$

The Order of Operations Agreement

The Order of Operations Agreement

Evaluate $2 + 3 \cdot 5$.

There are two arithmetic operations, addition and multiplication, in this problem. The operations could be performed in different orders.

Add first. $\underbrace{2 + 3} \cdot 5$

Then multiply. $\underbrace{5 \cdot 5}$

25

Multiply first. $2 + \underbrace{3 \cdot 5}$

Then add. $\underbrace{2 + 15}$

17

In order to prevent there being more than one answer to the same problem, an Order of Operations Agreement has been established.

The Order of Operations Agreement

THE ORDER OF OPERATIONS AGREEMENT

Step 1 Perform operations inside grouping symbols. **Grouping symbols** include parentheses ( ), brackets [ ], absolute value symbols | |, and the fraction bar.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

EXAMPLE

	$6 + 5(1 - 2)^4$
Perform operations inside grouping symbols (Step 1).	$= 6 + 5(-1)^4$
Simplify exponential expressions (Step 2).	$= 6 + 5(1)$
Do multiplication and division from left to right (Step 3).	$= 6 + 5$
Do addition and subtraction from left to right (Step 4).	$= 11$

Example 4

Simplify: $12 - 24(8 - 5) \div 2^2$

Solution:

$12 - 24(8 - 5) \div 2^2$

$= 12 - 24(3) \div 2^2$ Perform operations inside grouping symbols.

$= 12 - 24(3) \div 4$ Simplify exponential expressions.

$= 12 - 72 \div 4$ Do multiplication and division as they occur from left to right.

$= 12 - 18$

$= -6$ Do addition and subtraction as they occur from left to right.

The Order of Operations Agreement

When an expression has grouping symbols inside grouping symbols, first perform the operations inside the *inner* grouping symbols by following Steps 2, 3, and 4 of the Order of Operations Agreement.

Then perform the operations inside the *outer* grouping symbols by following Steps 2, 3, and 4 in sequence.