Real Numbers and Variable Expressions

CHAPTER

1

1.2 Operations with Integers

Objectives

1. Add Integers
2. Subtract Integers
3. Multiply Integers
4. Divide Integers
5. Application problems

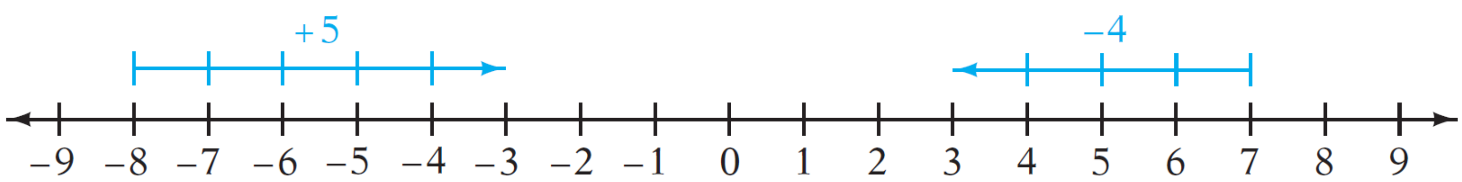
Add Integers

Add Integers

A number can be represented anywhere along the number line by an arrow.

A positive number is represented by an arrow pointing to the right, and a negative number is represented by an arrow pointing to the left.

The size of the number is represented by the length of the arrow.



Add Integers

Addition is the process of finding the total of two numbers. The numbers being added are called **addends.** The total is called the **sum.**

Addition of integers can be shown on the number line. To add integers, find the point on the number line corresponding to the first addend.

From that point, draw an arrow representing the second addend. The sum is the number directly below the tip of the arrow.

Add Integers

Following is the rules for adding integers.

ADDITION OF INTEGERS

Integers with the same sign

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

EXAMPLES

1. $2 + 8 = 10$ **2.** $-2 + (-8) = -10$

Integers with different signs

To add two numbers with different signs, find the absolute value of each number. Then subtract the smaller of these absolute values from the larger one. Attach the sign of the number with the larger absolute value.

EXAMPLES

3. $-2 + 8 = 6$ **4.** $2 + (-8) = -6$

Example 1

Add.

A. 162 + (–247) **B.** –14 + (–47) **C.** –4 + (–6) + (–8) + 9

Solution:

A. 162 + (–247) = –85

The signs are different. Subtract the absolute values of the numbers (247 – 162). Attach the sign of the number with the larger absolute value.

B. –14 + (–47) = –61

The signs are the same. Add the absolute values of the numbers (14 + 47). Attach the sign of the addends.

Example 1 – *Solution*

cont'd

C. $-4 + (-6) + (-8) + 9$

$= -10 + (-8) + 9$

$= -18 + 9$

$= -9$

To add more than two numbers, add the first two numbers. Then add the sum to the third number. Continue until all the numbers have been added.

Subtract Integers

Subtract Integers

Subtraction is the process of finding the difference between two numbers. Subtraction of an integer is defined as addition of the opposite integer.

SUBTRACTION OF INTEGERS

To subtract one integer from another, add the opposite of the second integer to the first integer.

EXAMPLES

	first number	−	second number	=	first number	+	the opposite of the second number		
1.	40	−	60	=	40	+	(−60)	=	−20
2.	−40	−	60	=	−40	+	(−60)	=	−100
3.	−40	−	(−60)	=	−40	+	60	=	20
4.	40	−	(−60)	=	40	+	60	=	100

Example 3

Subtract: –8 – 30 – (–12) – 7 – (–14)

Solution:

–8 – 30 – (–12) –7 – (–14)

= –8 + (–30) + 12 + (–7) + 14 Rewrite each subtraction as addition of the opposite.

= –38 + 12 + (–7) + 14 Add the first two numbers. Then add the sum to the third number. Continue until all the numbers have been added.

= –26 + (–7) + 14

= –33 + 14

= –19

Multiply Integers

Multiply Integers

Multiplication is the process of finding the product of two numbers. Several different symbols are used to indicate multiplication.

The numbers being multiplied are called **factors** and the result is called the **product.**

Note that when parentheses are used and there is no arithmetic operation symbol, the operation is multiplication.

Multiply Integers

MULTIPLICATION OF INTEGERS

Integers with the same sign

To multiply two numbers with the same sign, multiply the absolute values of the numbers. The product is positive.

EXAMPLES

1. $4 \cdot 8 = 32$ **2.** $(-4)(-8) = 32$

Integers with different signs

To multiply two numbers with different signs, multiply the absolute values of the numbers. The product is negative.

EXAMPLES

3. $-4 \cdot 8 = -32$ **4.** $(4)(-8) = -32$

Example 4

Multiply.

A. $-42 \cdot 62$ **B.** $2(-3)(-5)(-7)$

Solution:

A. $-42 \cdot 62 = -2604$

The signs are different. The product is negative.

B. $2(-3)(-5)(-7) = -6(-5)(-7)$

$= 30(-7)$

$= -210$

To multiply more than two numbers, multiply the first two numbers. Then multiply the product by the third number. Continue until all the numbers have been multiplied.

Divide Integers

Divide Integers

For every division problem there is a related multiplication problem.

Division: $\frac{8}{2} = 4$ Related multiplication: $4 \cdot 2 = 8$

Divide Integers

DIVISION OF INTEGERS

Integers with the same sign

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive.

EXAMPLES

1. $30 \div 6 = 5$ **2.** $(-30) \div (-6) = 5$

Integers with different signs

To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative.

EXAMPLES

3. $(-30) \div 6 = -5$ **4.** $30 \div (-6) = -5$

Divide Integers

Note that $\frac{-12}{3}$ = –4, $\frac{12}{-3}$ = –4, and $-\frac{12}{3}$ = –4.

This suggests the following rule.

If *a* and *b* are two integers, and *b* ≠ 0, then $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$.

Read *b* ≠ 0 as “*b* is not equal to 0.”

Divide Integers

The reason why the denominator must not be equal to 0 is explained in the following discussion of 0 and 1 in division.

ZERO AND ONE IN DIVISION

Zero divided by any number other than zero is zero.	$\frac{0}{a} = 0, a \neq 0$	because $0 \cdot a = 0$.
Any number other than zero divided by itself is 1.	$\frac{a}{a} = 1, a \neq 0$	because $1 \cdot a = a$.
Any number divided by 1 is the number.	$\frac{a}{1} = a$	because $a \cdot 1 = a$.
Division by zero is not defined.	$\frac{4}{0} = ?$	$? \times 0 = 4$ There is no number whose product with zero is 4.

EXAMPLES

1. $\frac{0}{7} = 0$

2. $\frac{-2}{-2} = 1$

3. $\frac{-9}{1} = -9$

4. $\frac{8}{0}$ is undefined.

Example 5

Divide.

A. (–120) ÷ (–8) **B.** $\frac{95}{-5}$ **C.** $-\frac{-81}{3}$

Solution:

A. (–120) ÷ (–8) = 15

The two numbers have the same sign.
The quotient is positive.

B. $\frac{95}{-5}$ = –19

The two numbers have different signs.
The quotient is negative.

C. $-\frac{-81}{3}$ = –(–27)

= 27

Application problems

Application problems

Let us find the *average* of all your test scores. You compute the average by calculating the sum of all your test scores and then dividing that result by the number of tests. Statisticians call this average an **arithmetic mean.**

Besides its application to finding the average of your test scores, the arithmetic mean is used in many other situations.

Example 6

The daily low temperatures, in degrees Celsius, during one week were recorded as follows: $-8°$, $2°$, $0°$, $-7°$, $1°$, $6°$, $-1°$. Find the average daily low temperature for the week.

Strategy:

To find the average daily low temperature:

- Add the seven temperature readings.
- Divide the sum by 7.

Example 6 – *Solution*

$$-8 + 2 + 0 + (-7) + 1 + 6 + (-1)$$

$$= -6 + 0 + (-7) + 1 + 6 + (-1)$$

$$= -6 + (-7) + 1 + 6 + (-1)$$

$$= -13 + 1 + 6 + (-1)$$

$$= -12 + 6 + (-1)$$

$$= -6 + (-1)$$

$$= -7$$

$$-7 \div 7 = -1$$

The average daily low temperature was –1°C.