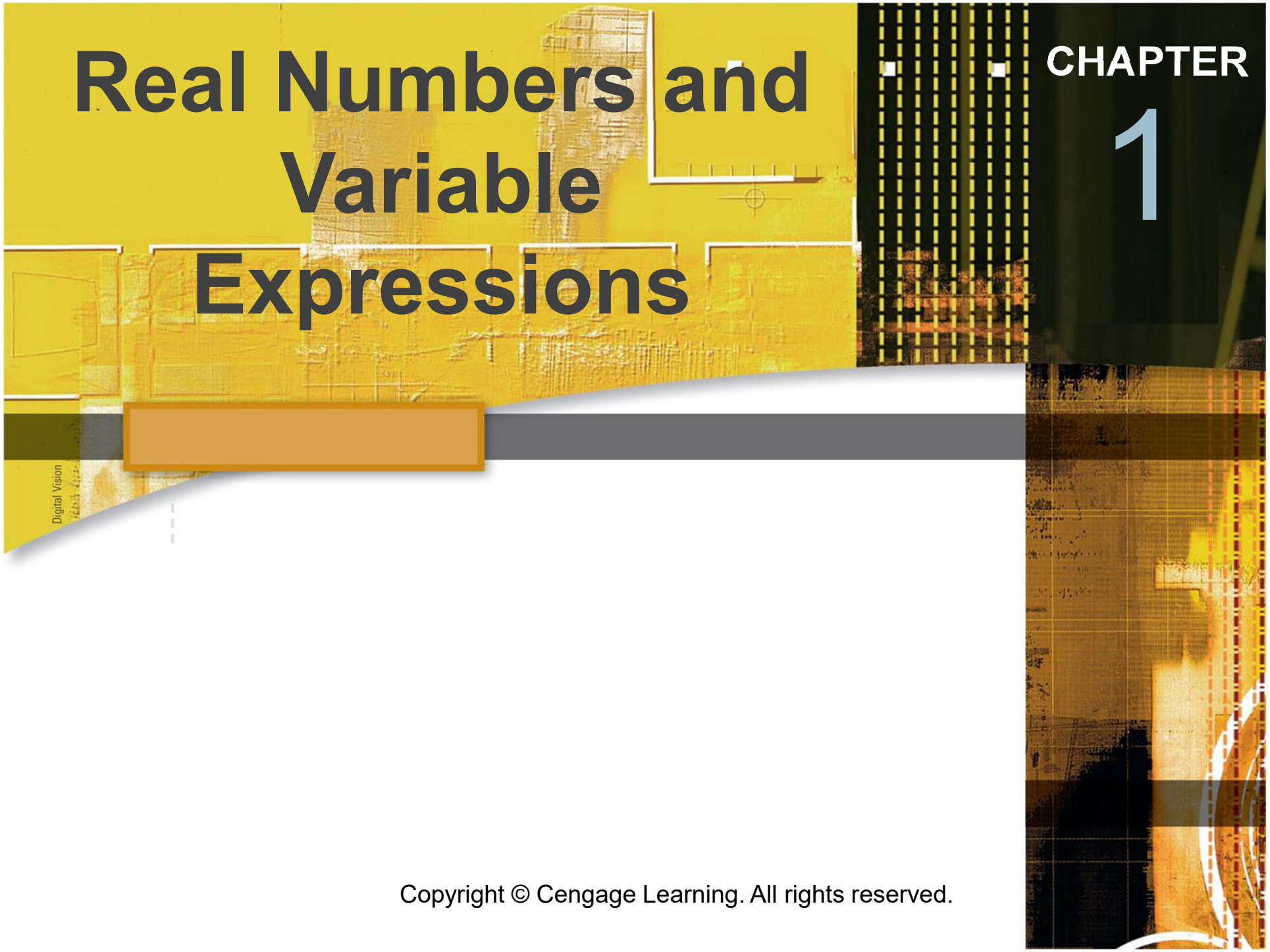
Real Numbers and Variable Expressions

CHAPTER 1



1.1 Introduction to Integers

Objectives

1. Order relations

2. Opposites and absolute value

Order relations

Order relations

Mathematicians place objects with similar properties in groups called sets.

A **set** is a collection of objects. The objects in a set are called **elements** of the set.

The **roster method** of writing sets encloses a list of the elements in braces.

The set of sections within an orchestra is written {brass, percussion, strings, woodwinds}.

Order relations

The numbers that we use to count objects, such as the number of students in a classroom or the number of people living in an apartment house, are the natural numbers.

Natural numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...}

The set of whole numbers includes the natural numbers and zero.

Whole numbers = {0, 1, 2, 3, 4, 5, 6, 7, ...}

Order relations

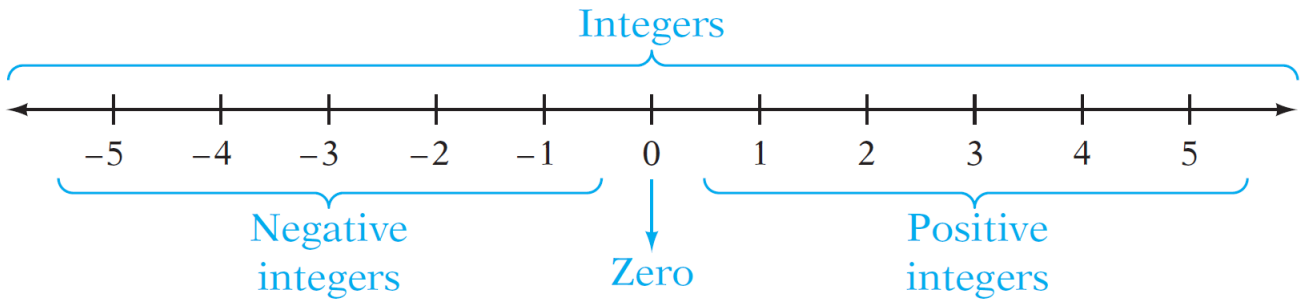
The whole numbers do not provide all the numbers that are useful in applications. For instance, a meteorologist also needs numbers below zero.

Integers = {... , –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}

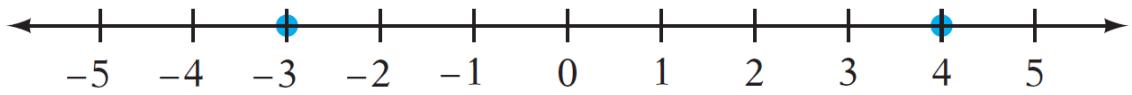
Each integer can be shown on a number line. The integers to the left of zero on the number line are called **negative integers.**

Order relations

The integers to the right of zero are called **positive integers** or natural numbers. Zero is neither a positive nor a negative integer.



The **graph of an integer** is shown by placing a heavy dot on the number line directly above the number. The graphs of –3 and 4 are shown on the number line below.



Order relations

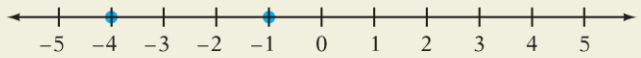
In mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable.** Variables are used in the following definition of inequality symbols.

DEFINITION OF INEQUALITY SYMBOLS

If a and b are two numbers, and a is to the left of b on the number line, then a **is less than** b. This is written $\boldsymbol{a < b}$.

EXAMPLES

1. $-4 < -1$ Negative 4 is less than negative 1.

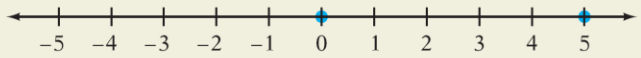


2. $-16 < -6$ Negative 16 is less than negative 6.

If a and b are two numbers, and a is to the right of b on the number line, then a **is greater than** b. This is written $\boldsymbol{a > b}$.

EXAMPLES

3. $5 > 0$ Five is greater than 0.



4. $-3 > -8$ Negative 3 is greater than negative 8.

Order relations

There are also inequality symbols for **is less than or equal to** (≤) and **is greater than or equal to** (≥).

$7 \leq 15$ 7 is less than or equal to 15.

This is true because $7 < 15$.

$6 \leq 6$ 6 is less than or equal to 6.

This is true because $6 = 6$.

Example 1

Use the roster method to write the set of negative integers greater than or equal to –6.

Solution:

$A = \{-6, -5, -4, -3, -2, -1\}$

A set is designated by a capital letter.
The roster method encloses a list of elements in braces.

Example 2

Given $A = \{-6, -2, 0\}$, which elements of set A are less than or equal to -2?

Solution:

$-6 < -2$

Find the order relation between each element of set A and -2.

$-2 = -2$

$0 > -2$

The elements -6 and -2 are less than or equal to -2.

Opposites and absolute value

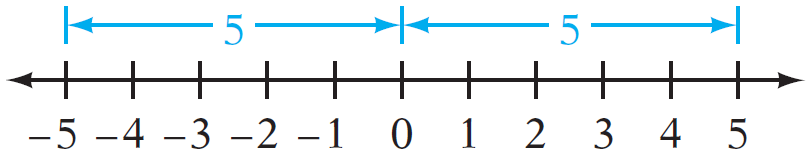
Opposites and absolute value

Two numbers that are the same distance from zero on the number line but are on opposite sides of zero are **opposite numbers** or **opposites.**

The opposite of a number is also called its **additive inverse.**

The opposite or additive inverse of 5 is –5.

The opposite or additive inverse of –5 is 5.



Opposites and absolute value

The negative sign can be read “the opposite of.”

$-(2) = -2$ The opposite of 2 is –2.

$-(-2) = 2$ The opposite of –2 is 2.

Example 3

Find the opposite number.

A. 6 **B.** –51

Solution:

A. The opposite of 6 is –6.

B. The opposite of –51 is 51.

Opposites and absolute value

The **absolute value** of a number is its distance from zero on the number line.

Therefore, the absolute value of a number is a positive number or zero. The symbol for absolute value is two vertical bars, | |.

ABSOLUTE VALUE

The absolute value of a positive number is the number itself. The absolute value of zero is zero. The absolute value of a negative number is the opposite of the negative number.

EXAMPLES

1. $|6| = 6$ **2.** $|0| = 0$ **3.** $|-6| = 6$

Example 4

Evaluate.

A. $|-4|$ **B.** $-|-10|$

Solution:

A. $|-4| = 4$

B. $-|-10| = -10$ The absolute value symbol does not affect the negative sign in front of the absolute value symbol. You can read $-|-10|$ as "the opposite of the absolute value of negative 10."