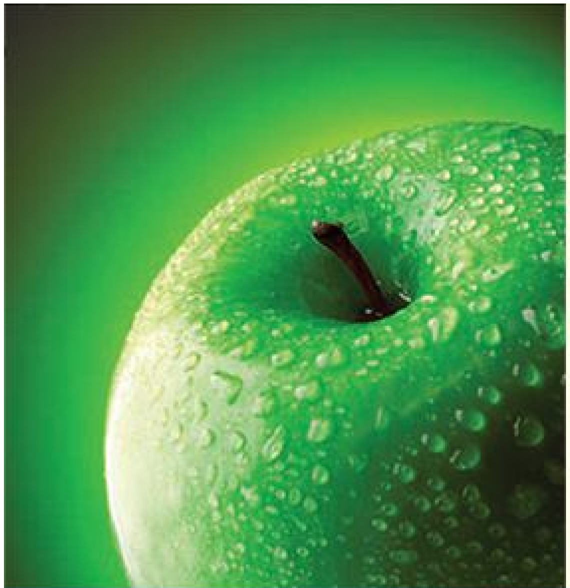


3 Understanding Addition and Subtraction

SECTION

3.2

Understanding Subtraction of Whole Numbers

What do you think?

- What does the term “borrow” mean in elementary school mathematics, and what does it mean in other contexts?

- Why do most current elementary texts discourage the term “borrow”?

- What other contexts for subtraction are there besides “take away”?

Understanding Subtraction of Whole Numbers

Subtraction Problems in Context

A. Write a word problem for 7 − 2 and draw a model that a young child might use to solve it before they knew subtraction.

B. Draw a model or use physical models for each of the following that a young child might use to solve the problem in the context.

1. Joe had 7 marbles. He lost 2 in a game. How many does he have left?

Understanding Subtraction of Whole Numbers

2. Billy has 2 marbles and Yaka has 7. How many more does Yaka have?

3. At the beginning of the week, a doctor had 7 ounces of insulin. During the week, 2 ounces of insulin were used. How much insulin did the doctor have at the end of the week?

4. You need \$7 to go to the movie and you only have \$2. How much more do you need?

5. Tom has 7 feet of bubblegum rope and Meg has 2 feet. How much more does Tom have?

Understanding Subtraction of Whole Numbers

Now go back to these five problems and answer the following questions:

- What action words describe what is happening in these five problems?
- How are the models/pictures of the problems similar and different?
- In what ways are the problems different? In what ways are they similar?
- What does subtraction mean? For example, what words come to mind when you think of subtraction?

Contexts for Subtraction

Contexts for Subtraction

In the above problems you likely had three different types of models/pictures, and some of them might have been drawn as a set model and others as a linear measurement model.

Most students, when they write a problem and when they think about subtraction, are thinking about the "**take-away**" context for subtraction.

In this context, we have some amount and we take away some amount, and our task is to figure out how much is left.

Contexts for Subtraction

Look back over the five problems in the above examples as well as the one you wrote and see which are “take-away.” What does the model/visual look like for these problems?

Problems 1 and 3 are take-away problems. It is likely that you wrote a take-away problem as well, as this is the most common one we think of with subtraction.

Think back to the previous section where we talked about adding discrete set models (collection of items) and measurement models.

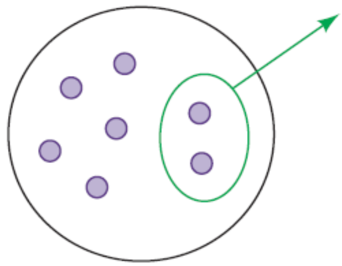
Contexts for Subtraction

We also have that distinction here where Problem 1 is a set model (because our items are sets of marbles) and Problem 3 is a measurement model (think about the markings of a measuring cup).

A child would likely solve the first one by representing the marbles (perhaps with actual marbles) and taking 2 away. A child might use a number line to solve Problem 3.

Let’s look at a take-away set model and a take-away linear model within these contexts.

Contexts for Subtraction



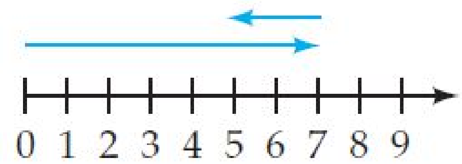
Take-away set model

Joe had 7 marbles. He lost 2 in a game. How many does he have left?

At the beginning of the week, the doctor had 7 ounces of insulin. During the week, 2 ounces of insulin were used.

Contexts for Subtraction

How much insulin did the doctor have at the end of the week?



Take-away linear measurement model

Another context for subtraction is when we are comparing two quantities and asking how much bigger one is than the other. Logically, we call this the "**comparison**" context for subtraction. Again, within this context we may have a set model or a linear measurement model.

Contexts for Subtraction

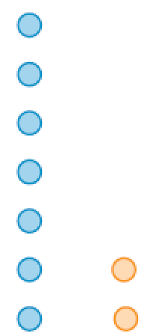
Look back at the problems and decide which ones have you comparing two quantities.

Problem 2 is a set model comparison context since you are comparing numbers of marbles (discrete items in a set).

Problem 5 is also a comparison model, but it is better represented on a number line as these are measurements. A child might draw or create the following to figure out the answers to each problem.

Contexts for Subtraction

Billy has 2 marbles and Yaka has 7. How many more does Yaka have?



Comparison set model

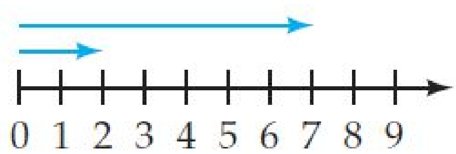
In this context, a child would likely lay out the 7 marbles (or some representation of them) that Yaka has and lay out the 2 that Billy has, and then count how many more Billy has.

Contexts for Subtraction

These models can help us see the difference between comparison and take-away. Compare the two models.

Now, let's model Problem 5 on a number line since it is a linear measurement model.

Tom has 7 feet of bubblegum rope and Meg has 2 feet. How much more does Tom have?



Comparison linear model

Contexts for Subtraction

Similar to the comparison set model, a child can now count up from 2 to 7 on the number line to answer the question.

The “**missing addend**” context is similar to the “comparison” model but is slightly different. We can think of this one as 2 + ____ = 7 (hence the name of missing addend).

In Problem 4, we might think of that as $2 plus how many dollars equals the $7 that we need.

Contexts for Subtraction

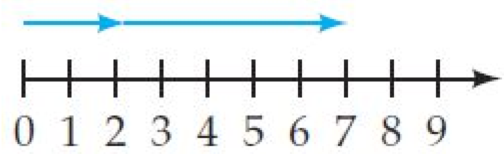
A child might answer this by starting with the $2 and adding on dollars until they reach the $7 needed.

What would a linear measurement model look like in this context? It would be similar to the dollars placed in a line, but the context would be a measurement.

Perhaps something like, we have 2 inches of ribbon but we need 7 inches to make a bracelet.

Contexts for Subtraction

How many more inches of ribbon do we need?



Here we start with the 2 inches and add on until we get to the 7 inches.

All of the above problems and models are 7 − 2 = 5, but they look very different. It is important mathematically that we not always think of subtraction as simply take-away, but be flexible to think about it in other contexts.

A Pictorial Model for Subtraction

A Pictorial Model for Subtraction

One way to express the commonality of all five problems just discussed is that in all cases, we have a large amount and two smaller amounts whose sum is equal to the larger amount.

We know the pictorial model for addition: If we invert that diagram, we have a pictorial representation of a general model for subtraction (Figure 3.6).

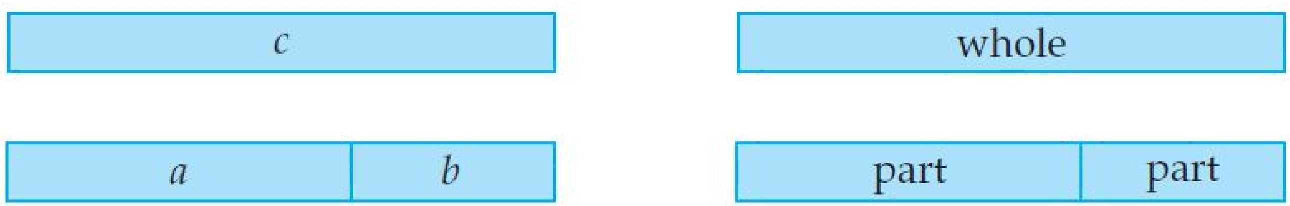


Figure 3.6

A Pictorial Model for Subtraction

These types of bar models are used extensively in Singapore, which produces some of the best math students in the world. We will use them in other places in the text, like with fractions and algebra.

It is important to note that *whole* and *part* refer to the numerical values rather than the contexts. For example, in Problem 2, we are comparing two wholes and saying that one whole contains 5 more marbles than the other.

A Pictorial Model for Subtraction

We define **subtraction** formally in the following manner:

$$c - b = a \quad \text{if} \quad a + b = c$$

That is, the difference between two numbers *c* and *b* is *a* if *c* is the sum of *a* and *b.*

A Pictorial Model for Subtraction

In mathematical language, *c* is called the **minuend**, *b* the **subtrahend**, and *a* the **difference**. This model also highlights the connections between addition and subtraction problems (Figure 3.7):

$$3 + 5 = 8$$

$$5 + 3 = 8$$

$$8 - 3 = 5$$

$$8 - 5 = 3$$

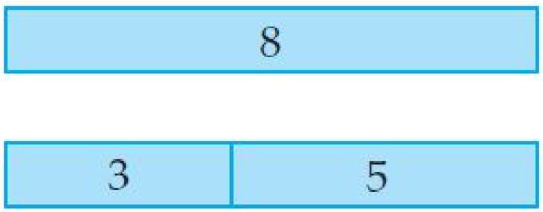


Figure 3.7

Properties of Subtraction

Properties of Subtraction

Think back to the properties of addition—identity, commutativity, associativity, and closure. Do those same properties hold for subtraction?

Some students think that subtraction has an identity property, because if we subtract zero from a number, its value does not change; that is, $a - 0 = a$.

This is true; however, if we reverse the order, the result is not true—that is, $0 - a \neq a$.

Properties of Subtraction

Therefore, we generally say that the operation of subtraction does not have an identity property.

After examining a few cases, you can see that the operation of subtraction does not possess the commutative property or the associative property.

The commutative property ($3 - 5$ does not equal $5 - 3$) is not immediately understood by children.

Properties of Subtraction

We recall one first-grader arguing that $3 - 5$ was 0 because "you can't have less than nothing" and another arguing that it was 2 because "you just turn them around."

Finally, the operation of subtraction is not closed for the set of whole numbers because the difference of two whole numbers can be a negative number, such as $3 - 5 = -2$ (which is not a whole number).

Investigation 3.2a – *Mental Subtraction*

Do the following computations in your head. Briefly note the strategies you used, and try to give names to them.

Note: One mental tool all students have is being able to visualize the standard algorithm in their heads.

For example, for the first problem, you could say, “Cross out the 6, replace it with 5, then add 10 to the 5 in the ones column, and think $15 - 8 = 7$ and then $5 - 2 = 3$; the answer is 37.”

Investigation 3.2a – *Mental Subtraction* continued

However, because we already know that method, we ask you not to use it here but to try others instead.

1. 65 − 28 **2.** 62 − 29 **3.** 184 − 125 **4.** 132 − 36
5. 1000 − 648

Investigation 3.2a – *Discussion*

As with addition, there are a variety of subtraction strategies. As we read the discussion below, reflect on the strategies and make sense of them in your mind. Nearly all of the strategies work better with certain kinds of numbers than with others.

Add up

In Problem 1, one alternative is to **add up**—that is, to ask how we get from 28 to 65.

Investigation 3.2a – *Discussion* continued

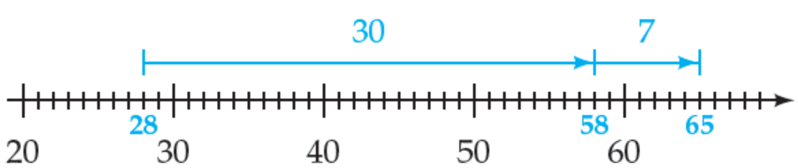
One student's actual strategy looks like this:

28 + 30 = 58

58 + 7 = 65

30 + 7 = 37, which is the answer.

The add-up strategy is nicely illustrated with a **number line**.



Investigation 3.2a – *Discussion* continued

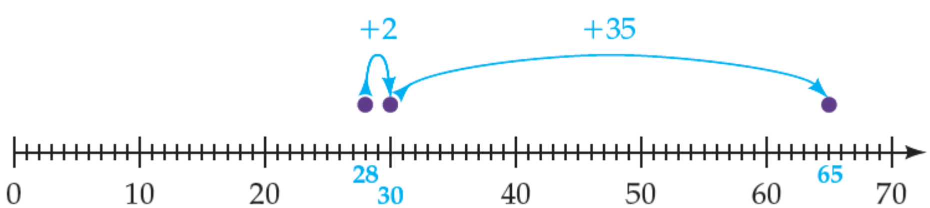
Another variation of adding up looks like this:

28 + 2 = 30

30 + 35 = 65

35 + 2 = 37, which is the answer.

This is illustrated on the number line.



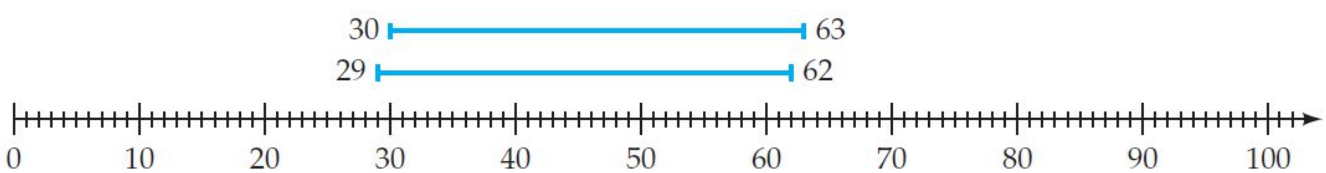
Investigation 3.2a – *Discussion* continued

Compensation

In Problem 2, we can use **compensation**, which does not work exactly the same way with subtraction as it does with addition.

This is how compensation looks with Problem 2: 62 − 29

Add 1 to both numbers. We now have 63 − 30, which is easily solved: 33.



Investigation 3.2a – *Discussion* continued

Some students understand the compensation strategy better by using a number line. On a number line, 62 − 29 can be interpreted as the distance between the two numbers. If we increase both numbers by 1, we have not changed the distance between the numbers.

In Problem 3, we can add up:

$$184 - 125$$

$$125 + 60 = 185$$

$$185 - 1 = 184, \text{ so the answer is } 59.$$

Investigation 3.2a – *Discussion* continued

In Problem 4, we can adapt the compensation strategy:

$$132 - 36$$

$$136 - 36 = 100.$$

Because we increased 132 by 4, we need to give back the 4, so the answer is $100 - 4 = 96$.

We can also solve Problem 4 by using **compatible numbers** and adding up:

$$132 - 36$$

$$36 + 64 = 100$$

$100 + 32 = 132$, so the answer is $64 + 32 = 96$.

Investigation 3.2a – *Discussion* continued

In Problem 5, we can use compatible numbers:

1000 − 648.

Realizing that 48 + 52 = 100, we can add up:

648 + 52 = 700

700 + 300 = 1000, so the answer is 300 + 52 = 352.

Investigation 3.2a – *Discussion* continued

Problem 5 can also be solved by adding up:

$1000 - 648$

$648 + 300 = 948$

$948 + 52 = 1000$, so the answer is $300 + 52 = 352$.

Children’s Strategies for Subtraction

Children's Strategies for Subtraction

Below are three very different methods that children have invented for subtracting larger numbers.

Let's explore these strategies with the problem $64 - 27$.

Break the second number apart

Essentially, this approach breaks apart the 27 and subtracts in pieces.

$$64 - 7 = 57$$

$$57 - 20 = 37$$

Children's Strategies for Subtraction

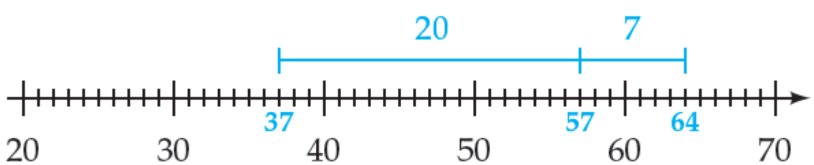
Do you see connection to any addition strategies?

This method is similar to adding up, where we begin with one number, break the second number into parts (expanded form), and add one place at a time. Here, we begin with the minuend and then subtract one place at a time.

As with addition, different representations illustrate and illuminate the problem in different ways.

Children's Strategies for Subtraction

On the number line, we move 7 units, which brings us to 57, and then we move 20 units, which brings us to 37. We can also represent the process numerically (below right).



$$
\begin{aligned}
& 64 - 27 \\
&= 64 - (20 + 7) \\
&= 64 - (7 + 20) \\
&= 64 - 7 - 20 \\
&= 57 - 20 \\
&= 37
\end{aligned}
$$

Children's Strategies for Subtraction

We can also break both numbers apart into tens and ones. This method works like this:

Subtract each place, beginning at the left. Next, combine the two differences: $40 - 3 = 37$.

$$\begin{array}{r} 64 \\ -27 \\ \hline \end{array}$$

$60 - 20 = 40$

$4 - 7 = -3$

$40 - 3 = 37$

Children's Strategies for Subtraction

Adding up

This method is very common, especially if the context of the problem is not take away but rather comparison or missing addend.

$$\begin{array}{r} 64 \\ \underline{-27} \end{array}$$

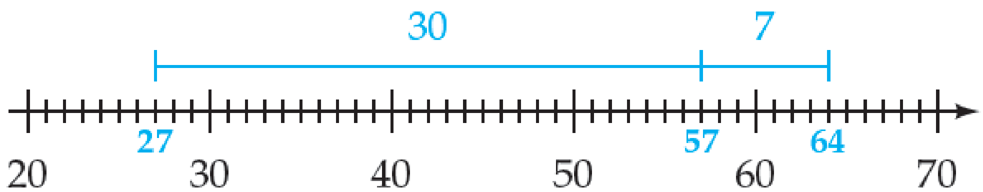
27 + 30 = 57

57 + 7 = 64

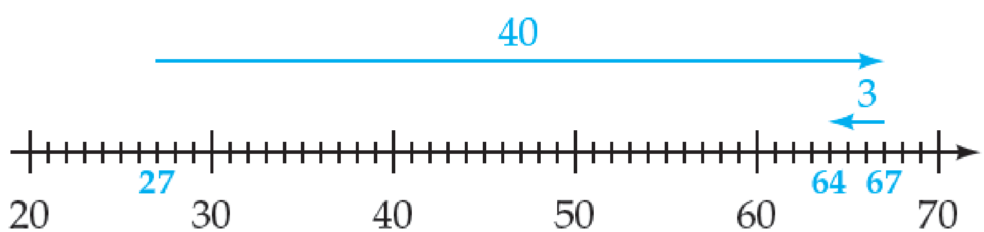
The answer is 30 + 7 = 37.

When asked to think out loud, the strategy sounds like this: "27 + 30 = 57 and 57 + 7 = 64, so the answer is 30 + 7 = 37."

Children's Strategies for Subtraction



Note that a common variation is to overshoot the number and then come back. For example, "27 + 40 = 67. Now we need to take away 3 to get to 64, so the answer is 40 − 3 = 37."



What if the numbers were bigger, for example 832 − 367? Look back on the approaches described previously.

How can you adapt them to this problem?

Investigation 3.2b – *Discussion*

"Break the second number apart" strategy extends but is now more cumbersome: Begin with the minuend and subtract each place.

$$\begin{array}{r} 832 \\ -367 \\ \hline \end{array} \quad \begin{array}{r} 832 \\ -7 \\ \hline 825 \end{array} \quad \begin{array}{r} 825 \\ -60 \\ \hline 765 \end{array} \quad \begin{array}{r} 765 \\ -300 \\ \hline 465 \end{array}$$

Using negative numbers extends quite well. In fact, many of the students actually find this strategy easier than the algorithm they grew up with.

$$\begin{array}{r} 832 \\ -367 \\ \hline \end{array} \quad \begin{array}{r} 800 \\ -300 \\ \hline 500 \end{array} \quad \begin{array}{r} 30 \\ -60 \\ \hline -30 \end{array} \quad \begin{array}{r} 2 \\ -7 \\ \hline -5 \end{array}$$

Investigation 3.2b – *Discussion* continued

$$500 - 30 = 470;\ 470 - 5 = 465$$

Adding up also becomes a bit cumbersome, but interestingly, some students feel quite comfortable with it. This is what they write:

$$\begin{array}{r} 832 \\ \underline{-367} \end{array}$$

367 + 3 = 370;

370 + 30 = 400;

400 + 432 = 832.

The answer is 3 + 30 + 432 = 465.

Investigation 3.2b – *Discussion* continued

Some children add up from left to right:

$367 + 400 = 767$; $767 + 60 = 827$; $827 + 5 = 832$.

The answer is $400 + 60 + 5 = 465$.

In the days before computerized cash registers, the cashiers used this method to count back change: If the bill was $3.80 and the customer used a $5 bill, the cashier would hand back two dimes and a dollar, saying 20 cents makes $4 and another dollar makes $5.

Understanding Subtraction in Base Ten

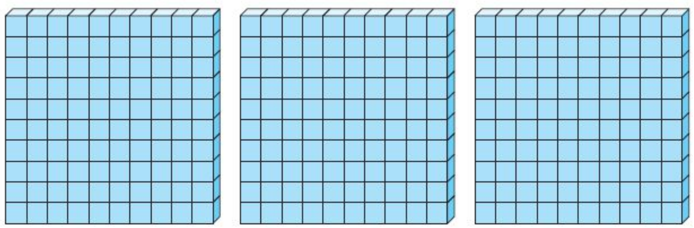
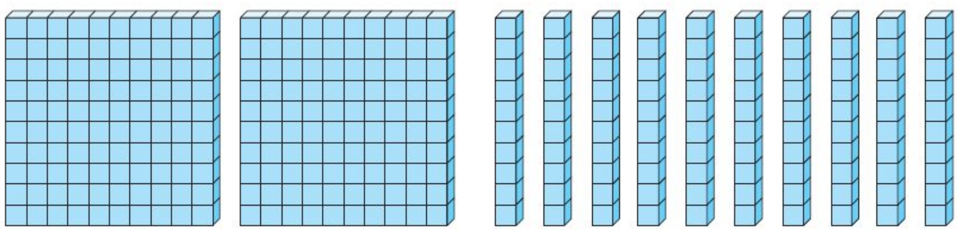
Understanding Subtraction in Base Ten

Let us now examine some standard and nonstandard algorithms for subtraction.

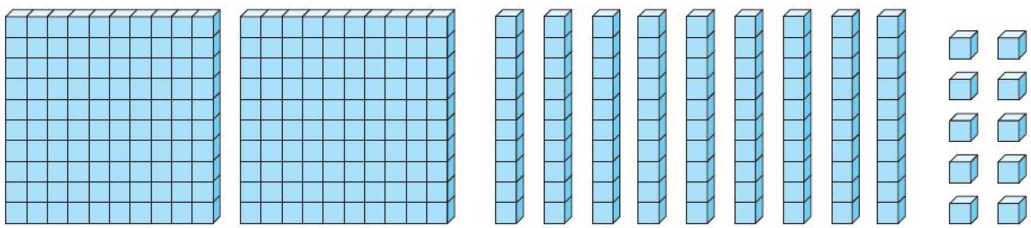
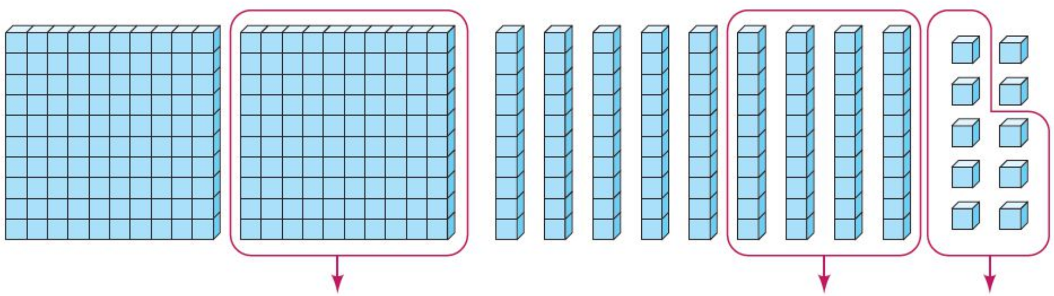
Researchers have found that when subtraction problems have zeros, the success rate for most third-graders goes down drastically. However, this need not be so. Let us examine the following problem: $300 - 148$.

Although there is no single procedure that all students use to solve this problem (unless they are forced to by their teacher), let's look at using a concrete model of base ten blocks, and see how that represents the traditional algorithm.

Understanding Subtraction in Base Ten

Using manipulatives	Using words	Using symbols
	We need to regroup 300 so that we can "take away" 148.	$\begin{array}{r} 3\;0\;0 \\ -1\;4\;8 \\ \hline \end{array}$
	We can trade 1 of the 3 hundreds for 10 tens, giving us 2 hundreds and 10 tens, which has the same value as 3 hundreds.	$\begin{array}{r} \overset{2}{\cancel{3}}\,{}^{1}0\;0 \\ -1\;4\;8 \\ \hline \end{array}$

Understanding Subtraction in Base Ten continued

Using manipulatives	Using words	Using symbols
	Next, we can trade 1 of the tens for 10 ones. We now have 2 hundreds, 9 tens, and 10 ones, which still has the same value as 3 hundreds.	$\begin{array}{r} \overset{2}{\cancel{3}}\ \overset{9}{\cancel{0}}\ {}^{1}0 \\ -\ 1\ \ 4\ \ 8 \\ \hline \end{array}$
	Now we can take away 1 hundred, 4 tens, and 8 ones.	$\begin{array}{r} \overset{2}{\cancel{3}}\ \overset{9}{\cancel{0}}\ {}^{1}0 \\ -\ 1\ \ 4\ \ 8 \\ \hline 1\ \ 5\ \ 2 \end{array}$

Justifying the Standard Algorithm

Justifying the Standard Algorithm

It is important to reemphasize that “standard” algorithms are not the “right” ones, or even the “best” ones, but rather the ones that, for various reasons, have become most widespread.

Many educators believe that more harm than good is done by forcing all students to learn the “standard” algorithms.

What is essential is that elementary students and teachers understand the algorithms that they use—that they understand the whys of the algorithm.

Justifying the Standard Algorithm

The base ten blocks help us to understand the algorithms, and it is important to make a direct connection between the written algorithms and what is happening with the blocks.

Some students find that representing the problem in expanded form is helpful. A key to understanding this algorithm is to understand the equivalence of 3 hundreds and 2 hundreds, 9 tens, and 10 ones.

Justifying the Standard Algorithm

Although they look different, and the numerals are different (300 versus $29^{1}0$), the amounts are equal. Another key to understanding this algorithm is to know why we needed to do the trading.

This is easier to see at the concrete level; without trading, we cannot literally "take away" 1 flat, 4 longs, and 8 singles.

$$
\begin{array}{l}
\ \ 3 \text{ hundreds} + 0 \text{ tens} + 0 \text{ ones} \\
-1 \text{ hundred } \ + 4 \text{ tens} + 8 \text{ ones} \\
\hline
\text{Cannot subtract the tens or} \\
\text{the ones}
\end{array}
\rightarrow
\begin{array}{l}
\ \ 2 \text{ hundreds} + 10 \text{ tens} + 0 \text{ ones} \\
-1 \text{ hundred } + \ 4 \text{ tens} + 8 \text{ ones} \\
\hline
\text{Cannot subtract the ones} \\
\
\end{array}
\rightarrow
\begin{array}{l}
\ \ 2 \text{ hundreds} + 9 \text{ tens} + 10 \text{ ones} \\
-1 \text{ hundred } \ + 4 \text{ tens} + 8 \text{ ones} \\
\hline
\text{Subtraction possible} \\
\
\end{array}
$$

Investigation 3.2c – *Subtracting in Base Five*

What if we had a base five system instead of a base ten system? How would the standard algorithm for subtraction look different, and how would it be the same?

Use models (snap cubes work well if you have them), pictures, or virtual manipulatives to figure out the answer to the following.

If you Google "virtual manipulatives," the first site listed is the National Library of Virtual Manipulatives. Under Number & Operations, there are two links that will work for this problem.

Investigation 3.2c – *Subtracting in Base Five* continued

One is called Base Blocks where you can start with the first number and take away (by throwing in the trash can icon) what you want to subtract.

The other is called Base Blocks Subtraction, which is a comparison model where you put the top number in blue and what you want to subtract in red and put one on top of the other to make them disappear.

Make sure you change the base to 5!

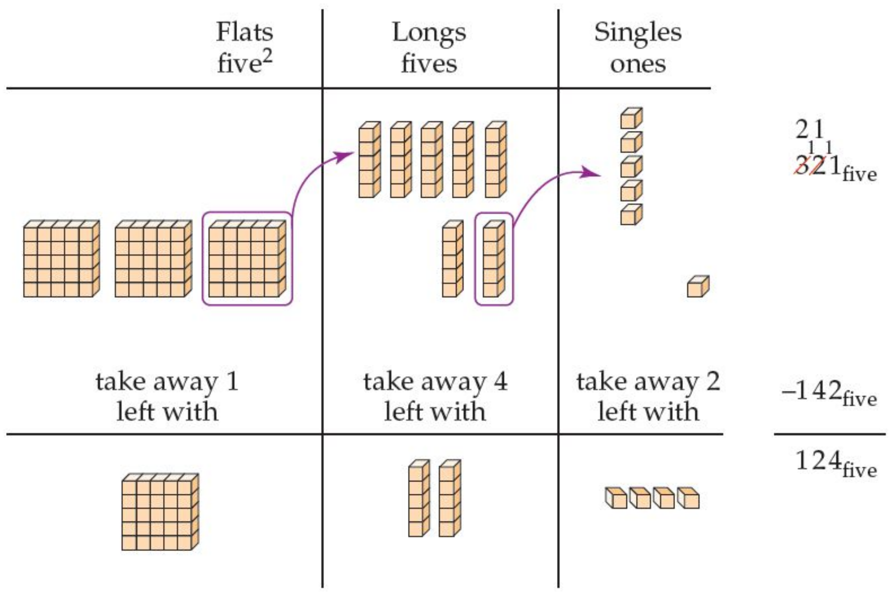
Investigation 3.2c – *Subtracting in Base Five* continued

To move blocks from left to right, drag them over and they will break apart in the next column.

$$\begin{array}{r} 321_{\text{five}} \\ -\,142_{\text{five}} \\ \hline \end{array}$$

Investigation 3.2c – *Discussion*

We will use a “take-away” model, since that is what we did in the previous base ten example. Examine the picture below and see if you can figure it out for yourself; then we will explain.



Investigation 3.2c – *Discussion* continued

We begin by drawing 3 flats (five by five), 2 longs (five), and 1 single (one).

If we start in the ones place as the standard algorithm does, we see that we cannot take away 2 so we need to trade a long for five singles as shown here with the arrow.

Now we can take away the 2 singles and are left with 4 singles.

Now we have 1 long and want to take away 4 longs, so we have to trade 1 flat for 5 longs.

Investigation 3.2c – *Discussion* continued

Then we take away 4 longs and are left with 2 longs. Finally we take away the 1 flat and get our final answer of 124_{five}.

How is this different than in base ten? The only difference is in the trading. Instead of trading ten, we trade five.

Thinking about the algorithms in other bases helps us to understand them better in base ten. You will likely never teach your students to subtract in base five, but it helps you develop a deeper understanding of place value and the algorithms that you memorized in elementary school.

Investigation 3.2d – *An Alternative Algorithm*

People who grew up in many European countries, Mexico, and others learned a very different subtraction algorithm.

Below are two examples of its use and what a person using this algorithm would say.

For the first problem, “You can’t take 8 from 4, so you put a 1 next to the 4, cross out the 6, and put a 7. Now subtract: 14 − 8 = 6, 8 − 7 = 1, 9 − 3 = 6.”

Investigation 3.2d – *An Alternative Algorithm* continued

For the second problem, “You can’t take 8 from 3, so you put a 1 next to the 3, cross out the 5, and put a 6.

Now, you can’t take 6 from 2, so you put a 1 next to the 2, cross out the 1, and put a 2. Now subtract: 13 − 8 = 5, 12 − 6 = 6, 6 − 2 = 4.”

$$
\begin{array}{r}
984 \\
-\,368 \\
\hline
\end{array}
\qquad
\begin{array}{r}
98{}^{1}4 \\
{}^{7} \\
-\,3\cancel{6}8 \\
\hline
616
\end{array}
\qquad
\begin{array}{r}
623 \\
-\,158 \\
\hline
\end{array}
\qquad
\begin{array}{r}
6{}^{1}2{}^{1}3 \\
2\ 6 \\
-\,\cancel{1}\cancel{5}8 \\
\hline
465
\end{array}
$$

Investigation 3.2d – *Discussion*

Can you explain why this algorithm works? That is, can you justify it, or prove it?

The justification for this algorithm is very different from the justification for the previous algorithm, in which the trading didn't change the value of the minuend, but only its representation.

Here, however, the value of both the minuend and the subtrahend *do* change: The value of the minuend and the subtrahend are now 994 and 378.

Investigation 3.2d – *Discussion* continued

Make sure you see this before reading on. Now look at the two number lines below. Can you explain now why this algorithm works?

Because the value of both minuend and subtrahend have increased by 10, the difference (distance) between them remains the same. We added 10 ones to the 984 and we added 1 ten to the 368.

Justifying the Standard Algorithm

The discussion of these two algorithms is a nice place to introduce a framework for examining algorithms that was presented by Hyman Bass in the February 2003 issue of *Teaching Children Mathematics.*

Bass proposes that we examine five qualities of an algorithm:

- *Accuracy* (or reliability). The algorithm should always produce a correct answer.
- *Generality*. The algorithm applies to all instances of the problem, or class.

Justifying the Standard Algorithm

- *Efficiency* (or complexity). This refers to whether the cost (the time, effort, difficulty, or resources) of executing the algorithm is reasonably low compared to the input side of the problem.

- *Ease of accurate use* (vs. proneness to error). The algorithm can be used reasonably easily and does not lead to a high frequency of error in use.

- *Transparency* (vs. opacity). What the steps of the algorithm mean mathematically, and why they advance toward the problem solution, is clearly visible.

Justifying the Standard Algorithm

Let us use this framework to examine the traditional algorithm, the alternative algorithm, and the negative numbers algorithm.

Accuracy. Used correctly, all of these algorithms will always produce a correct answer.

Generality. All the algorithms will work with any subtraction problem.

Efficiency. The traditional algorithm doesn't work exactly the same in all cases.

Justifying the Standard Algorithm

The alternative algorithm also requires some additional thought if there are 9s in the subtrahend. The negative numbers algorithm works identically in all situations.

Ease of accurate use. In certain situations children are more likely to make errors with algorithms. The negative numbers strategy requires stronger computation skills because of the use of negative numbers.

Justifying the Standard Algorithm

Transparency. The traditional algorithm is more transparent (especially if there is a visual representation like the base ten blocks connected to the written algorithm), which is a primary reason why it supplanted the alternative algorithm in the United States in the twentieth century. It is easier for children to understand the “why” of the traditional (you don’t change the value) than the “why” of the alternative (because both increase by the same amount, the difference is still the same). The negative numbers strategy is also transparent.

Investigation 3.2e – *Children’s Mistakes in Subtraction*

There is a very powerful video in *Integrating Mathematics and Pedagogy to Illustrate Children’s Reasoning* that shows a second-grade child not understanding why $70 - 23$ is not equal to 53.

How do you think she got 53? What misunderstanding(s) of place value and/or subtraction might be causing her problems?

Investigation 3.2e – *Discussion*

$$\begin{array}{r} 70 \\ -23 \\ \hline 53 \end{array}$$

In the video, you hear her say that "0 take away 3 is 3," which is a common misconception among young children.

Interestingly, even when the interviewer had the child do the problems with base ten blocks and a hundred chart, even though she got the correct answer using both of those representations, she trusted the answer she got using pencil-and-paper more!

Investigation 3.2f – *Rough and Best Estimates with Subtraction*

In each of the problems below, first obtain a quick rough estimate. Then obtain the best estimate you can.

A. In 2018, Acstead State College had an enrollment of 4234, whereas Milburn College had an enrollment of 3475. How many more students did Acstead have than Milburn?

$$\begin{array}{r} 4234 \\ -\,3475 \\ \hline \end{array}$$

B. The attendance at the Yankees game was 73,468, whereas the attendance at the Mets game was 46,743. How much greater was the attendance at the Yankees game?

$$\begin{array}{r} 73{,}468 \\ -\,46{,}743 \\ \hline \end{array}$$

Investigation 3.2f – *Discussion*

A. You can get a quick estimate by rounding:

$$4200 - 3500 = 700.$$

To get a more accurate estimate, you could use adding up:

3475 + 800 = 4275, then back off 40 to get 4235.
The estimate is 800 − 40 = 760.

B. Leading digit: 7 − 4 = 3, giving an estimate of about 30,000.

Investigation 3.2f – *Discussion* continued

Rounding: 70,000 − 50,000 = 20,000.

Rounding and adding up: 47,000 + 3000 makes 50,000.
Adding up to 73,000 gives 23,000.

Our estimate is 3000 + 23,000 = 26,000.

Investigation 3.2g – *Number Sense with Subtraction*

A. Fill in the blanks to make this subtraction problem.

$$\begin{array}{r} \square\,\square\,\square\,\square \\ -\,4\,2\,6\,3 \\ \hline 3\,1\,5\,9 \end{array}$$

Investigation 3.2g – *Discussion*

At first glance, some students panic. “There is no way I can do this problem!” However, if we stop for a moment and remember the relationship between addition and subtraction, we can see that we can quickly determine the missing number by simply adding 3159 + 4263!

A. Take no more than 5 to 10 seconds to determine whether this answer is reasonable. That is, make your determination, using number sense, without doing any pencil and-paper work.

$$\begin{array}{r} 6542 \\ -\,2847 \\ \hline 3125 \end{array}$$

Investigation 3.2g – *Discussion* continued

For most people the simplest way to proceed here is to add 3125 and 2847 and see if the answer is close to 6542.

A quick estimation of the leading two digits shows 3100 + 2900 = 6000, so the answer is not true.